THE CONFLUENT SYSTEM FORMALISM:
II. THE GROWTH HISTORY OF OBJECTS

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ABSTRACT

With the present paper we conclude the presentation of a semianalytical model of hierarchical clustering of bound virialized objects formed by gravitational instability from a random Gaussian field of density fluctuations. In paper I, we introduced the basic tool, the so-called "confluent system" formalism, able to follow the evolution of bound virialized objects in the peak model. This was applied to derive the mass function of objects. In the present paper, we calculate other important quantities characterizing the growth history of objects. This model is compared with a similar one obtained by Lacey & Cole (1993) following the Press & Schechter (1974) approach. The interest of the new modeling presented here is twofold: 1) it is formally better justified as far as peaks are more reasonable seeds of objects than the indetermined regions in the Press & Schechter prescription, and 2) it distinguishes between merger and accretion enabling us to unambiguously define the formation and destruction of objects and to estimate growth rates and characteristic times not available in the previous approach.

Subject headings: cosmology: theory – galaxies: clustering – galaxies: formation

1. INTRODUCTION

Observational data on the distant universe are increasingly numerous and detailed. Comparison with the nearby universe harbors important information on the formation and evolution of cosmic objects. However, our capability of extracting accurate cosmological implications from these observations is severely limited by the lack of an analytical model of gravitational clustering. This is a fundamental lack only partially replaced by numerical simulations.

In the most studied scenario of structure formation via gravitational instability from a primordial random Gaussian field of density fluctuations, Press & Schechter (1974; PS) derived a practical analytical estimate for the number density of bound virialized objects (or more exactly, steady relaxed haloes) of mass M at any given epoch. This mass function has been shown to agree with N-body simulations (Nolthenius & White 1987; Efstathiou et al. 1988; Efstathiou & Rees 1988; Carlberg & Couchman 1989; White et al. 1993; Bahcall & Cen 1993; Lacey & Cole 1994). But the mass function of objects does not provide all the information required in many cosmological problems. The rates at which objects grow and the characteristic times of this process are also needed (e.g., Toth & Ostriker 1992; Richstone, Loeb, & Turner 1992; Kauffman, White & Guiderdoni 1993; Lacey

& Cole 1993; González-Casado, Mamon, & Salvador-Solé 1994; Kauffman 1994; González-Casado et al. 1996). Richstone, Loeb, & Turner (1992) used the time evolution of the PS mass function to estimate the formation rate of objects of mass M at different epochs. However, this is not a very accurate estimate since the time derivative of the mass function is equal to the rate at which objects reach mass M *minus the rate at which they leave this state*, both terms having comparable values.

Following the PS original prescription or a better sound version of it using the excursion set formalism, Bower (1991) and Bond et al. (1991) derived the mass function of objects of a given mass at a given epoch subject to the condition of being part of another object with a larger mass at a later time. This conditional mass function was used by Lacey & Cole (1993, LCa; see also Kauffmann & White 1993) to infer self-consistent estimates of the instantaneous merger rate and the typical age and survival time of objects. This clustering model has been shown by Lacey & Cole 1994 (LCb) to agree with N-body simulations. However, as recognized by these authors, there is the formal caveat that the PS approach on which it is based is heuristic. In particular, the seeds of objects are indetermined, possibly even unconnected, regions. As a consequence, one cannot really count objects with

a given mass, but just calculate the probability that a given point is in an object of that mass, which is at the base of a slight inconsistency in the analytical estimate of the formation time for power spectra with index $n > 0$ (see LCa).

But there is perhaps a more serious problem with the PS approach. This deals with one unique process of instantaneous mass increase of objects, their capture of other more or less massive ones, process which is generically called merger. Although strictly correct, this viewpoint is at the base of an important shortcoming. For usual power spectra of the initial density field, the number density of objects diverges at small masses. Consequently, objects continuously experience captures, there being no specific event in this approach marking the beginning or the end of any stable entity. The age and survival time of an object must then be defined rather *arbitrarily* in terms of some relative mass variation along the series of embedded objects connecting with it. Moreover, these definitions are *ambiguous* since they do not account for the fact that the same mass increase can be achieved in a smooth quasi-continuous manner or essentially in one unique capture at some intermediate time.

This is particularly disturbing since the notions themselves of the formation and destruction of objects have been introduced with the aim to specify when the instantaneous mass increase of a system is important enough to appreciably affect its structural properties. In this sense, the distinction between "tiny" and "notable" captures would be very convenient. Kitayama & Suto (1996) have recently proposed a modified version of the LCa model in this spirit. However, the PS approach is not well adapted to such a distinction which makes the model of Kitayama & Suto be somewhat inconsistent (see § 2 for details). But this does not imply, of course, that such a distinction cannot be self-consistently achieved in any other approach.

In tiny captures, the internal kinetic and potential energies of the capturing system are not appreciably modified from their equilibrium values. So the system remains essentially unaltered despite the slight extra mass added. In notable captures (possibly the same preceding events but considered from the viewpoint of the partner object), there is on the contrary a marked departure in the internal kinetic and/or potential energies of the capturing system. This causes an important rearrangement of the system as it relaxes approaching the virial relation. We can therefore natu-

rally say that, in the former case, the final and initial relaxed systems are the same object (or equivalently, that the object survives to the capture) while, in the latter, they constitute two distinct objects. In other words, we can define accretion as those tiny captures contributing to the (quasi)continuous mass increase of objects, and mergers as those notable ones destroying and giving them rise. Objects are then well-defined as those stable entities surviving, while smoothly accreting mass, between two mergers marking their formation and destruction, and the age and survival time of objects would have an obvious *unambiguous* meaning.

The problem with this viewpoint is, of course, where to place the division between tiny and notable captures. Establishing whether there is a significant rearrangement of the system as a consequence of a capture is equivalent to establishing whether the capture causes to it a significant departure from steadiness. So the difficulty in making that division is similar and, in fact, tightly connected with that encountered in making the distinction between relaxed and non-relaxed objects (or in identifying the exact extent of the former ones): deciding whether a region is steady or not. As far as objects are continuously capturing other more or less massive ones strict steadiness is never achieved. Consequently, making *in practice* any of these decisions requires choosing some confidence level for assessing the significance of the departure from steadiness, which introduces some arbitrariness. However, the advantage of working with *theoretical models* is that they do not deal with the full complexity of the real universe but with some suitable idealization of it. For example, the modeling of gravitational clustering presumes it is possible to tell, in some unspecified manner, between relaxed and non-relaxed systems. So why could it not also presume it is possible to tell, in a similar unspecified manner, between accretion and merger in the sense above?

As shown in paper I (Manrique & Salvador-Solé 1995; see also Salvador-Solé & Manrique 1995), making these two sole assumptions in the peak model framework leads to one unique, well justified, self-consistent, clustering model. The basic tool is a new formalism, the so-called confluent system of peak trajectories, able to follow the filtering evolution of peaks tracing the clustering evolution of objects (Salvador-Solé & Manrique 1994). This formalism was applied, in paper I, to derive the mass function of objects. In the present paper, we apply it to calculate their

growth rates and times. This clustering model is similar to that developed by LCa in the sense that these quantities are derived from the statistics of the continuous random density field encountered at a fixed arbitrary epoch after recombination when fluctuations are still linear (with the growing factor only a function of time at all interesting scales) and Gaussian distributed, and the dynamics of dissipationless collapse are approximated by the spherical model. However, it is physically better motivated since the assumed seeds of objects are peaks instead of the indetermined regions in the PS approach. In addition, it allows us to *unambiguously* define the formation and destruction of objects and calculate their characteristic growth rates and times. It is important to remark that there is no freedom in the model; there are only one or two parameters (depending on the power spectrum) governing the dynamics of collapse which can be adjusted by fitting the real mass function of objects. In particular, there is *no arbitrariness* associated with the distinction between merger and accretion.

But what about the connection between this idealized model and real (simulated or observational) data? As mentioned, the practical distinctions between relaxed and non-relaxed and between tiny and notable captures require the choice of two, likely connected, confidence levels. Now, if the model is good enough it will agree with empirical data *for some specific values* or some narrow ranges of values of these confidence levels or any equivalent pair of parameters, such as the total mean overdensity of the system, $\langle \rho \rangle / \rho$ with ρ the mean cosmic density, and the relative mass increase of the system $\Delta M/M$ (both in principle functions of M and t), for the distinctions between relaxed and non-relaxed objects and between merger and accretion, respectively. These would be the unspecified manners of carrying out those distinctions implicitly assumed in the model. This is the scheme followed by the PS approach for dealing with the non-trivial problem of defining and identifying relaxed objects. The model here proposed is intended to similarly deal with the problem of defining and identifying the formation and destruction of relaxed objects.

The predicted mass function of relaxed objects agrees with the PS one recovering the results of N-body simulations for relaxed haloes identified as those regions with overdensity $\langle \rho \rangle / \rho$ greater than a universal threshold (for $\Omega = 1$) equal to 178. Cole & Lacey (1996) have shown, indeed, that this corresponds to

the maximum extent of regions in steady state at some reasonable (unspecified) confidence level. The validity of our model concerning the predicted growth rates and times is harder to check because no practical distinction has ever been attempted to implement in numerical simulations between accretion and merger. Nonetheless, a first hint can be provided by the comparison of our results with those obtained from the LCa model also checked to agree with numerical simulations.

In § 2 we remind the basic lines of the confluent system formalism. In § 3 we derive the instantaneous formation, destruction, and mass accretion rates of objects. Their typical ages and survival times are calculated in § 4. Our results are discussed and summarized in § 5. The notation used in this paper is the same as in paper I. Since many calculations are based on results obtained by Bardeen et al. (1986; BBKS) we have kept as close as possible to the notation introduced by these authors. The main difference comes from the fact that we deal with three different kinds of densities, while BBKS only dealt with one. Especial caution must be made in not mixing them up. Firstly, there are the normal and conditional density functions of peaks *at a fixed filtering scale R* per infinitesimal ranges of ν, i.e., the density contrast δ scaled to the rms value $\sigma_0(R)$, and other possible variables. These were already defined in BBKS and are denoted by a caligraphic capital n just as in that paper. A minor difference with the notation used in BBKS is that we specify the fixed value of the filtering scale R as one parameter. For example, we write $\mathcal{N}(\nu, R)\, d\nu$ instead of $\mathcal{N}(\nu)\, d\nu$. Second, there are the normal and conditional density functions of peaks *at a fixed density contrast δ* per infinitesimal ranges of the filtering scale R and other possible variables. These density functions, already introduced in paper I, are denoted by a roman capital n with the fixed value of δ as one parameter. We write, for example, $N(R, \delta)\, dR$. Finally, and this is a novelty, there are also density functions per infinitesimal ranges of both R and δ (or the corresponding mass M and time t, respectively) and any other extra variable. These are denoted by a capital n in boldface, for example, $\mathbf{N}(R, \delta)\, dR\, d\delta$. These different symbols, $\mathcal{N}$, N, and $\mathbf{N}$, are usually accompanied by one superindex specifying the characteristic property (if any) of the peaks involved and the subindex pk or no subindex at all depending on whether all peaks with that characteristic property are included or just those of them tracing bound viri-

alized objects (see § 2), respectively. Hereafter, we assume comoving units.

2. THE CONFLUENT SYSTEM FORMALISM

By assumption, a merger is any *discontinuous* mass increase along the temporal series of relaxed systems subtending at each step the preceding one. (In practice, there must be "some significant discontinuity in mass", or equivalently "some significant structural rearrangement", or still "some significant departure from steadiness" between two consecutive relaxed systems in the discrete, fine enough, temporal series one can consider.) When an object merges we say that it is destroyed. Note that if an object is captured by any more massive it automatically merges regardless of whether or not this more massive object survives to the event. If it does not we say that the final system subtending the mass of the merging objects is a new object that has just formed.

In contrast, accretion is any *continuous* (we assume it also derivable) mass increase along the temporal series of relaxed systems subtending at each step the preceding one. (In practice, if there is "no significant discontinuity in mass", or "no significant rearrangement", or still "no significant departure from steadiness" along the series of discrete fine steps considered, one can regard the mass increase as achieved in a continuous and derivable manner.) We say that an object survives as long as it evolves by accretion. When an object evolving by accretion is captured by any more massive object it merges, hence it is destroyed. However, an object evolving by accretion can capture less massive objects without being destroyed. In fact, its (continuous) mass increase is made at the expend of very tiny objects. Only if they capture smaller although notable objects (yielding a significant discontinuity in its mass increase) they also merge and are destroyed. This is the kind of mergers mentioned above giving rise to the formation of new objects.

Notice that these definitions of accretion and merger refer to the object whose evolution is being followed, not to the event itself. This latter can be regarded either as merger or as accretion depending on the object under consideration. The inconsistency of the model proposed by Kitayama & Suto (1996) arises from not accounting for this fact. These authors take the model of LCa and make the extra assumption that any object which results from another one increasing its mass by less than a factor 2 can be "identified" to it. This kind of capture is therefore regarded as

accretion, while that in which the mass increases by more than a factor 2 is regarded as a true merger. The instantaneous formation/destruction rate of objects of mass M is then defined as the intantaneous total merger rate for final/initial objects of that mass *including only* those captures of objects less/more massive than half/two-times the mass M. Now, according to the definitions adopted for accretion and merger, all these captured objects merge indeed in the event, while those more/less massive than half/two-times the mass M only accrete. However, not all those mergers cause the formation of new objects of mass M as presumed; a large fraction necessarily contributes to the mass increase *by accretion* of those objects identified to the final ones with mass M.

Coming back to our model, the peak model ansatz states that there is a correspondence between peaks of fixed linear overdensity in the filtered density field at some arbitrary epoch t_i and relaxed objects at the time t. The overdensity δ_c is assumed to be a decreasing function $\delta_c(t)$ of the collapse time, and the filtering scale R an increasing function $R(M)$ of the mass of the resulting objects. Given the assumed distinction above between accretion and merger for real relaxed objects, that correspondence automatically leads to the natural identification and distinction between each other of accretion and merger events *in the filtering process*. A peak on scale $R + \Delta R$, with ΔR positive and arbitrarily small, is the result of the evolution by accretion of a peak on scale R provided only that the volume (mass) subtended by the latter is essentially embedded within the one subtended by the former. (Strictly, we should talk about volumes subtended by "collapsing clouds associated with peaks", i.e., the regions surrounding each peak which enclose a total mass equal to that of the corresponding final object at t. But, for simplicity, we will hereafter say the volumes subtended by peaks. Likewise, we will say "nested peaks" instead of "peaks with nested associated collapsing clouds".) Whenever the identification between couples of peaks on contiguous scales is not possible, that is, whenever there is a discrete jump in scale between two consecutive embedded peaks, we are in the presence of a merger. It is important to mention that the filtering of the density field must be carried out with a Gaussian window for the density contrast δ of peaks to diminish with increasing filtering scale R as required by consistency with the growth in time of the mass of objects. Hereafter we adopt this particular window.

From this identification among peaks on different scales we have that when an object evolves by accretion its associated evolving peak traces a continuous and derivable trajectory $\delta(R)$ in the δ vs. R diagram. (Note that despite the fact that the density field is assumed continuous and infinitely derivable, peak trajectories are only required to be one time derivable as the assumed mass increase in time of accreting objects.) In contrast, when an object merges the evolving peak tracing it becomes nested on a larger scale peak with identical δ, which yields a discrete horizontal jump in scale of the associated peak trajectory in the δ vs. R diagram. Therefore, to compute the density of objects at t in an infinitesimal range of masses dM we must calculate the density of *non-nested* peaks with fixed value of δ appropriate to t on scales in the infinitesimal range dR corresponding to dM. This density, $N(R, \delta)\, dR$, can be obtained from the density of peaks satisfying identical constraints although disregarding whether they are nested or not, $N_{pk}(R, \delta)\, dR$, and the density of these same peaks subject to the condition of being located in a background with the same density contrast δ on a different filtering scale R', $N_{pk}(R, \delta | R', \delta)\, dR$. Indeed, these three quantities are related through

$$N(R, \delta) = N_{pk}(R, \delta)$$
$$- \int_R^\infty dR'\, \frac{M(R')}{\rho}\, N(R', \delta)\, N_{pk}(R, \delta | R', \delta), \quad (1)$$

with $M(R)$ the inverse relation to $R(M)$. Notice that this relation is approximated for it assumes the whole cloud associated with the peak on scale R' with uniform smoothed density contrast equal to its central value.

Equation (1) is a Volterra type integral equation of the second kind for the unknown function $N(R, \delta)$. This can be obtained by means of the standard iterative process from the known functions $N_{pk}(R, \delta)$ and $N_{pk}(R, \delta | R', \delta)$, respectively equal to $\mathcal{N}_{pk}(\nu, R)$ $\langle x \rangle\, \sigma_2\, R/\sigma_0$ and $\mathcal{N}_{pk}(\nu, R | \nu', R')\, \widetilde{\langle x \rangle}\, \sigma_2\, R/\sigma_0$ in terms of the normal and conditional density functions of peaks at a fixed scale R per infinitesimal range of $\nu = \delta/\sigma_0$, calculated by BBKS. In this latter expressions σ_i are the i-th order spectral moments, which only depend on R, and $\langle x \rangle$ and $\widetilde{\langle x \rangle}$ are some averages of $x \equiv -(\sigma_2\, R)^{-1} \partial_R \delta$, which depend on R and δ as well as on R' in the latter case, defined in paper I. The mass function of objects at t is then simply

$$N(M, t) = N(R, \delta_c)\, \frac{dR}{dM}, \quad (2)$$

with the dependence on M and t on the right hand side given by the functions $R(M)$ and $\delta_c(t)$. As shown in paper I, very general consistency arguments allow one to determine the shape of these two functions as

$$R(M) = \frac{1}{\sqrt{2\pi}\, q} \left(\frac{M}{\rho} \right)^{1/3}, \quad (3)$$

$$\delta_c(t) = \delta_{c0}\, \frac{a(t_i)}{a(t)} = \delta_{c0} \left(\frac{t_i}{t} \right)^{2/3} \quad (4)$$

with a the cosmic scale factor and q and δ_{c0} two constants governing the exact dynamics of collapse. For CDM and the $n = -2$ power-law spectra, a good fit is obtained to the PS mass function for q equal to ~ 1.45 in both cases (in the power law case, the larger n, the smaller q) and δ_{c0} equal to ~ 6.4 and 8.4, respectively. (As mentioned in paper I, there is no degeneracy in the present formalism between these two parameters. It is therefore not surprising that a value of q appreciably different from unity is coupled to a value of δ_{c0} so different from the standard value of 1.686.) Strictly, equations (3) and (4) are only valid for the scale-free case, i.e., an Einstein-de Sitter universe ($\Omega = 1$, $\Lambda = 0$) and a power-law spectrum. However, as shown in paper I, they are good effective relations for other power spectra such as the CDM one. Furthermore, following the same strategy as in LCa, it should be possible to extend the applicability of the model to the cases $\Omega \neq 1$ and/or $\Lambda \neq 0$. We want to emphasize that the values of parameters q and δ_{c0} quoted above, and used throughout the present paper to illustrate the practical implementation of the model, correspond to those giving acceptable fits to the original PS mass function (for the standard top hat window and critical threshold density) for the same cosmogonies. This fitting tends to privilege the small mass end while the model will be finally applied rather to massive objects. Thus, finer values of these parameters should be inferred by directly fitting N-body simulations in the relevant scales.

Given the characterization of accretion and merger, it is clear that the density of objects being destroyed (because merging) in a given interval of time is given by the density of non-nested peaks which become nested along the corresponding decrement in δ. The density of forming objects is not so simple to obtain. We first need to characterize those filtering events which contribute with the *appearance* of new peaks. As just mentioned, when an object merges the non-

nested peak tracing its evolution in the δ vs. R diagram experiences a discrete jump in the scale. But this does not mean, of course, that every non-nested peak partaking of the same event necessarily experiences this kind of jump. The largest scale peak will the most often just keep on evolving in a continuous manner. (This reflects the well-known fact, in gravitational clustering, that a merger from the viewpoint of one given object can be seen as accretion for the most massive partner.) But it will sometimes happen that the largest scale peak also experiences a discrete jump. Then, it will not be possible to identify the final non-nested peak with any of its ancestors. This appearance of a new non-nested peak therefore traces the formation of a new bound object. Hence, the density of objects forming (because of some mergers) in an interval of time should be given by the density of peaks appearing in the sense above along the corresponding decrement in δ.

Actually, things are a little more complicated than this. The continuous trajectory attached to an accreting non-nested peak can be suddenly truncated (it is not possible to identify the peak at the current δ with any peak on an infinitesimally larger scale) without becoming nested. In addition, peaks can not only become nested into larger scale ones but they can also sporadically leave their hosts. According to the correspondence between relaxed objects and non-nested peaks, the preceding events can only be interpreted as tracing a relaxed object breaking up in small pieces and a relaxed object spalling off another object, respectively. Thus, we are concerned with 5 different filtering processes: accretion (continuous non-nested peak trajectories), aggregation (continuous non-nested peak trajectories becoming nested into old or newly appeared continuous non-nested peak trajectories), fusion (appearance of continuous non-nested peak trajectories), and the inverse processes of the latter two, namely, spallation (continuous nested peak trajectories becoming non-nested from surviving or just disappeared continuous non-nested peak trajectories), and fission (disappearance of continuous non-nested peak trajectories). Since spallation and fission are clearly unrealistic processes from the gravitational viewpoint, the confluent system formalism cannot be used to follow the clustering of *individual* objects. However, our purpose is not to follow the evolution of individual objects, but to own a good *statistical* description of the general clustering process.

Therefore, what exactly traces, statistically, the amount of objects that are destroyed (because merging) or that form (as the result of some mergers) along some interval of time Δt is the *net* decrease in old non-nested peak trajectories (given by the amount of aggregations minus spallations) and the *net* increase in new non-nested peak trajectories (given by the amount of fusions minus fissions) yielded along the corresponding $-\Delta\delta$, respectively. Hereafter, we will simply refer to these quantities as the net density of peaks becoming nested and the net density of non-nested appearing peaks, respectively. To calculate the former we must compute the density of peaks at δ which in continuously evolving (without fissioning) to $\delta - \Delta\delta$ are found to be nested, minus the density of peaks at δ which are already nested and do not fission in the next $-\Delta\delta$. Indeed, this latter term includes peaks which can end up, at $\delta - \Delta\delta$, being nested or non-nested. Hence, that difference really gives the density of non-nested peaks at δ which have just become nested through aggregations, minus the density of nested peaks at δ which have just become non-nested through spallations, as wanted. (We have required all peaks at δ not to be fissionable because aggregation and spallation yield, by definition, continuous peak trajectories.) Similarly, to calculate the latter net density above we must compute the density of non-nested peaks at δ which do not fission in the next $-\Delta\delta$, minus the density of peaks at δ which have continuously evolved (without fissioning) from peaks at $\delta + \Delta\delta$. Indeed, this latter term includes peaks which at δ become fissionable or not. Hence, that difference really gives the density of non-nested peaks at δ that have just appeared through fusions, minus the density of non-nested peaks at δ that are just going to disappear (or, in the limit of vanishing $\Delta\delta$, have just disappeared before reaching δ) through fissions, as wanted. (We have required all the peaks at δ to be non-nested because fusion and fission only concern, by definition, peaks which are non-nested in appearing or disappearing.) Consequently, what we only need for the model to be *statistically* acceptable is those two net quantities tracing the densities of objects merging and forming along some interval of time be positive.

Let us finally note that mass is usually not conserved in individual filtering mergers (more exactly, in aggregations or spallations). Indeed, the mass of an aggregated (spalled) peak does not yield any instantaneous finite change in the mass of its partner.

Only in the case of fusions (fissions) will the mass of the final (initial) peak be close to that of all aggregating (spallating) peaks. In any event, the total mass of peaks partaking of the filtering processes tracing the mergers of objects is certainly conserved *statistically*, as needed. Indeed, the correct normalization at every δ of the scale function of non-nested peaks (see paper I) and the fact that the total available mass in any representative (large enough) comoving volume is fixed guarantee that the the mass shared by peaks in that volume is preserved under any instantaneous rearrangement through aggregations and spallations and the fusions and fissions they experience.

3. GROWTH RATES

As just mentioned, the density of objects that are destroyed (because merging) in the interval dt is traced by the net density of peaks becoming nested in the corresponding range $-d\delta$. To be more exact, we are interested in calculating the net density of peaks with δ_c on scales between R and $R + dR$ which become nested into (i.e., aggregate into, minus spallate off) non-nested peaks with $\delta_c - d\delta$ on scales between R' and $R' + dR'$, $\mathbf{N}^d(R \to R', \delta_c)\, dR\, dR'\, d\delta$. This net density is derived in Appendix B. Now, by dividing it by $N(R, \delta_c)\, dR$ we obtain the net conditional probability that a non-nested peak with δ on scale R becomes nested into a non-nested peak with $\delta_c - d\delta$ on scales between R' and $R' + dR'$. And from this conditional probability we can readily infer the instantaneous destruction (or true merger) rate at t for objects of mass M per specific infinitesimal range of mass M' ($M < M'$) of the resulting objects,

$$r^d(M \to M', t) = \frac{\mathbf{N}^d(R \to R', \delta_c)}{N(R, \delta_c)}\, \frac{dR'}{dM'} \left| \frac{d\delta_c}{dt} \right|, \quad (5)$$

with R, R', and δ_c on the right hand side written in terms of M, M', and t, respectively, through equations (3) and (4). (Note that this destruction rate involves any possible merger that an object can experience with other objects, disregarding whether or not this yields the formation of a new object.)

This destruction (or true merger) rate is plotted in Figure 1 for different masses M of the initial object and t equal to the present time. As can be seen, it shows a very different behavior from the merger rate obtained by LCa: the former vanishes while LCa's diverges at small $\Delta M/M$. This marked difference simply reflects the different viewpoints adopted in the two models. In the PS approach, any mass increase

of objects in any given interval of time is necessarily due to (generic) mergers, while in ours it can be due to mergers or accretion. So both merger rates cannot be identical. For small intervals of time, large mass increments are predominantly caused by mergers rather than accretion. Therefore both models should yield similar merger rates at very large $\Delta M/M$ as found. Conversely, very small mass increments should tend to become accretion. In fact, we would even expect they all become accretion for $\Delta M/M$ below any effective threshold marking the border line between tiny and notable captures. So it is well understood that our merger rate vanishes at very small $\Delta M/M$ while LCa's diverges there because of the divergent number of infinitesimal mass captures. (Note that what LCb compared with N-body simulations is not their merger rate, but the conditional probability that given an object of fixed mass at some initial epoch it is incorporated into a larger mass object at a later epoch. Since this quantity does not depend on the particular definition adopted for merger and accretion, the good agreement found by these authors between theory and simulations does not favor LCa's viewpoint as compared to ours.)

Our merger rate therefore shows the good expected behavior at both mass ends. Moreover, the sharp cutoff shown at small $\Delta M/M$ points to a division between accretion and merger roughly consistent with a fixed effective threshold in $\Delta M/M$ (at least for given M and t). However, at relatively large values of $\Delta M/M$, our merger rate is significantly lower than LCa's which is hard to interpret in terms of our distinction between accretion and merger. In fact, this effect becomes increasingly marked as we diminish M: a central dip develops finally reaching negative values. So not only does the correct behavior of the model seem to require large M but there is a strict lower bound in M for its general physical consistency. (Negative merger rates imply more spallations than aggregations which is unrealistic in hierachical clustering.) The value of this lower bound in M is not dramatic by itself. At present, it is of the order of the mass of small dwarf galaxies ($\sim 2.5 \times 10^9\ M_\odot$ and $\sim 2.5 \times 10^8\ M_\odot$, for the CDM and $n = -2$ power-law spectra here analized, respectively) and it increases with increasing time so that it encompasses all relevant cosmological scales ($\sim 1.8 \times 10^6\ M_\odot$ and $\sim 1.1 \times 10^6\ M_\odot$) at a redshift of just 1.25. What is disturbing is the own existence of this bound and the related unsatisfactory behavior of the merger rate at
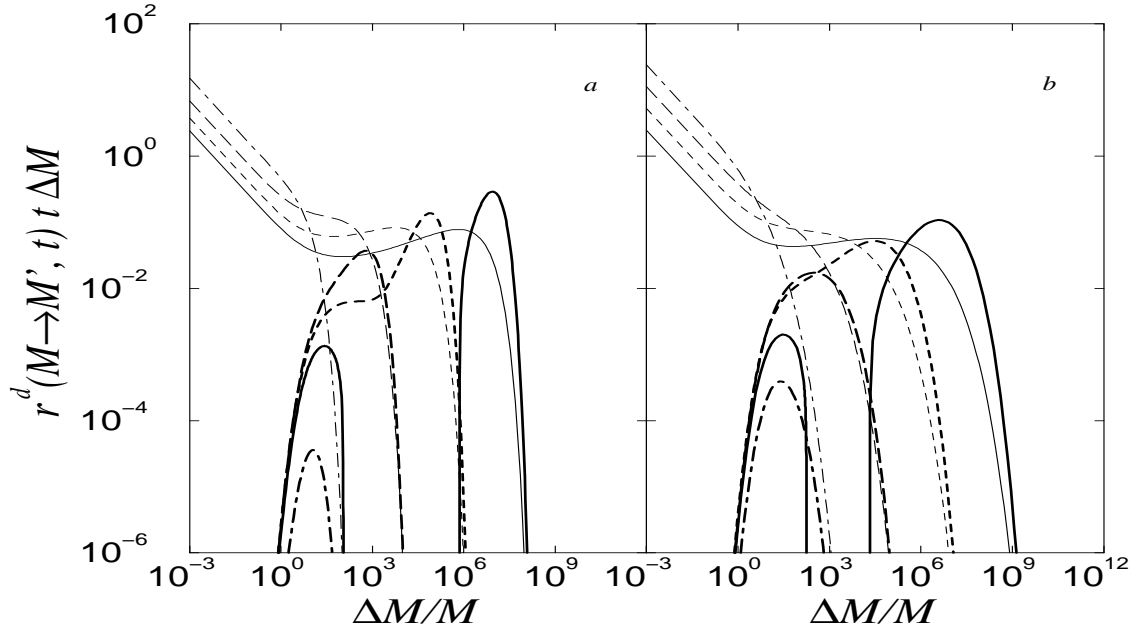
Fig. 1.— Instantaneous destruction (or true merger) rate at the present time for an object with mass $M/M_\odot$ equal to 10^8 (solid lines), 10^{10} (dashed lines), 10^{12} (long-dashed lines), and 10^{14} (dotted-dashed lines) as a function of the mass ΔM of the merger partner for the CDM (a) and $n = -2$ power-law (b) power spectra, both normalized to $\sigma(8\,h^{-1}\,Mpc) = 0.67$ in an Einstein-de Sitter universe with $h = 0.5$. In thin lines the merger rates drawn from the LCa model.

moderate $\Delta M/M$. These shortcomings do not seem to be inherent to the basic assumption of the model, that is, the possibility to effectively distinguish between accretion and merger. They could be partly due to the fact that the peak model is only a good approximation for the gravitational growth of massive objects. (At any given δ_c, the larger the scale, the higher the peak amplitude as compared to the typical fluctuation, $\sigma_0(R)$. And since the higher the peak, the more spherical its shape and the more negligible the shear caused by the surrounding matter, the better its gravitational collapse is described by the spherical approximation.) But what the most likely causes them is the approximation used in equation (1). Indeed, some preliminary calculations seem to indicate that a more accurate approximation for the nesting correction should notably improve the performances of the model. In the following, we will not insist anymore on the limited domain of consistency of the present model (in its current version) and focus on the analysis of its predictions for large enough M.

From the confluent system formalism making the natural distinction between accretion and merger we can also calculate the transition rate *including both kinds of captures*, mergers and accretion. Equation

(5) gives the rate at which objects of mass M merge and are destroyed giving rise to objects of mass M'. But from the viewpoint of these latter objects the event can be seen either as accretion, if they can be identified with the initial most massive partner, or as a true merger, if they cannot. Thus, the instantaneous accretion+merger, or simply capture, rate for final objects of mass M' per specific infinitesimal range of the captured mass M ($M < M'$) is

$$r^c(M' \leftarrow M, t) = \frac{r^d(M \to M', t)\, N(M, t)}{N(M', t)}, \quad (6)$$

with r^d given by equation (5). This capture rate is not directly comparable to the LCa merger rate plotted in Figure 1 since it refers to the mass M' of the final object instead of to the mass M of the initial object. But it can be readily compared to the capture rate drawn from the LCa merger rate through the same relation (6). This comparison is shown in Figure 2. Once again we find a markedly different behavior between the solution of the two models. This difference is also due to the distinct viewpoint adopted in the two models. The capture rate obtained from our model making the distinction between accretion and merger only includes, in the case of accretion, those objects
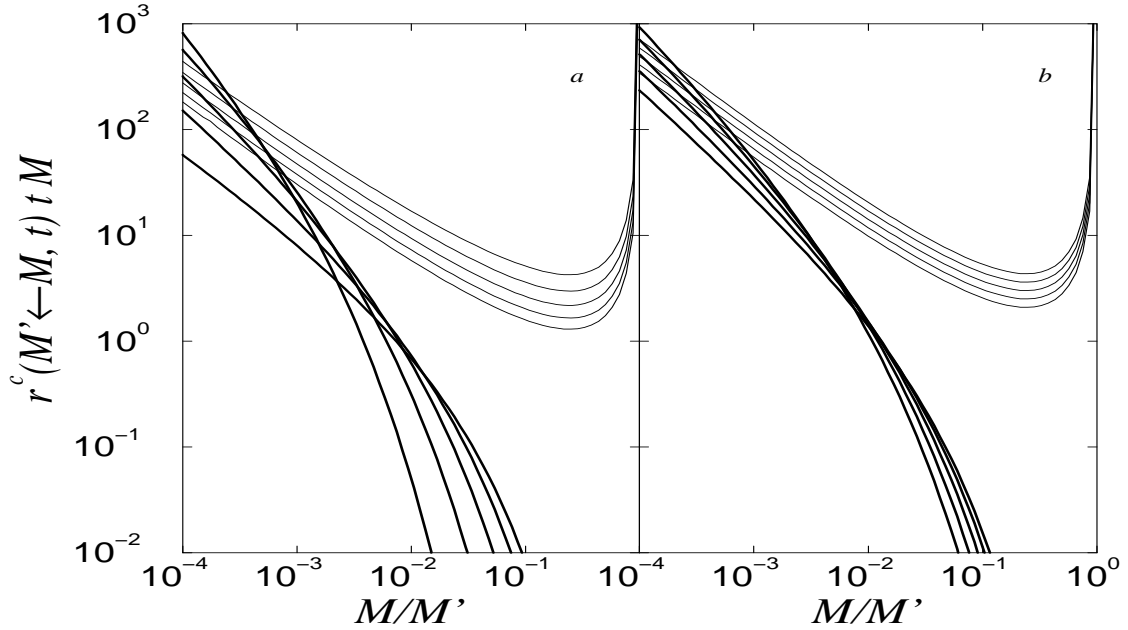
Fig. 2.— Instantaneous capture (or accretion+ merger) rate at the present time for a final object with mass $M'/M_\odot$ equal to 10^{14} (the lowest curve on the left), 3×10^{13}, 9×10^{14}, 2.7×10^{15}, and 8.1×10^{15} (the highest curve on the left) as a function of the mass M of the captured objects for the same cosmogonies (a) and (b) as in Fig. 1. In thin lines the capture rates inferred from the LCa merger rates.

which are accreted, obviously not the accreting ones identified to the final objects of mass M', and hence, not considered as captured by them. In contrast, all objects are counted in the LCa capture rate (in particular, those seen as accreted as well as those seen as accreting from our point of view) because there is no identification in the LCa model between the final and any of the initial objects partaking of any capture. This clearly explains the difference between the two capture rates at captured masses M close to M'. On the other hand, the two models should show a similar behavior at small M, in full in agreement with what is found.

The previous destruction and capture rates are per *specific* infinitesimal range of mass of the final or captured objects, respectively. To derive the respective *global* rates we must simply integrate equation (5) over M'

$$r^d(M,t) = \int_M^\infty r^d(M \to M',t)\,dM', \qquad (7)$$

and equation (6) over M

$$r^c(M',t) = \int_0^{M'} r^c(M' \leftarrow M,t)\,dM. \qquad (8)$$

Note that these global rates are obviously positive down to much smaller masses M or M' than the respective specific ones for the whole range of the extra variable. It is worthwhile mentioning that these global merger and capture rates cannot be compared with the respective quantities in the LCa model because these cannot be calculated in that model (the corresponding integrals diverge). This is an important drawbak of the PS approach related to the fact no distinction is made between merger and accretion.

Let us now turn to the formation rate. The density of objects that form (as the result of mergers) in the interval of time dt is traced by the net density of non-nested appearing peaks yielded in the corresponding range $-d\delta$. More exactly, we are interested in calculating the net density of non-nested peaks with δ_c on scales between R and $R+dR$ that just appear (i.e., fusion minus fission) in the preceding range $-d\delta$, $\mathbf{N}^f(R,\delta_c)\,dR\,d\delta$. This net density is derived in Appendix C. Now, by dividing it by $N(R,\delta_c)\,dR$ we are led to the net conditional probability that a non-nested peak with scale R appears between δ and $\delta_c - d\delta$. Therefore, the instantaneous formation rate
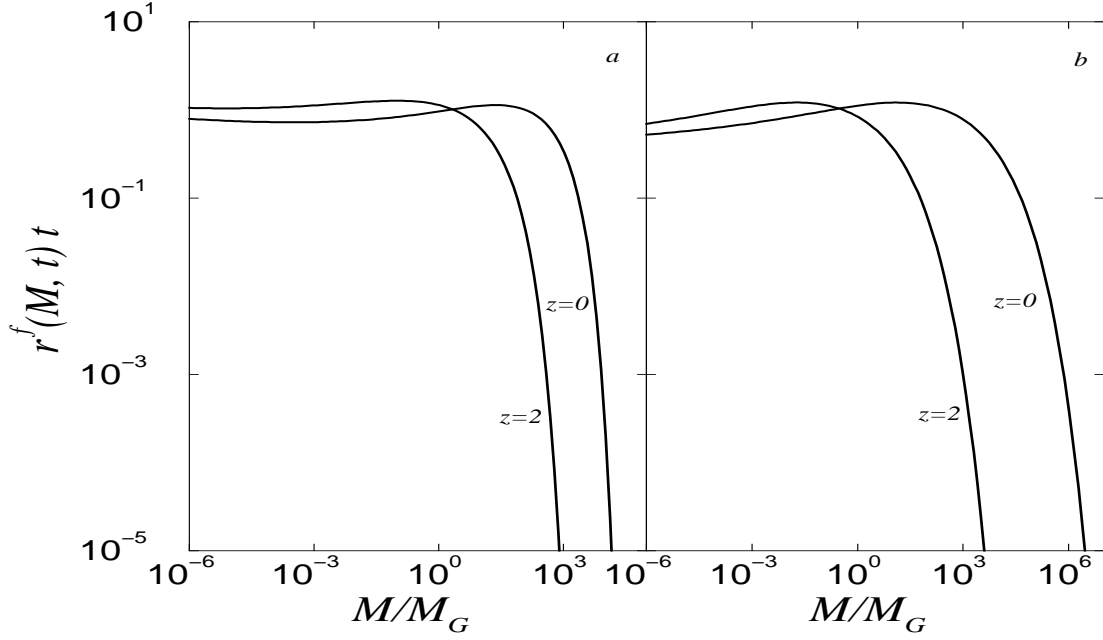
Fig. 3.— Instantaneous formation rate at two different epochs as a function of the mass M (in units of $M_G = 10^{12}\, M_\odot$) of objects for the same cosmogonies (a) and (b) as in Fig. 1.

at t of objects of mass M is

$$r^f(M,t) = \frac{\mathbf{N}^f(R,\delta_c)}{N(R,\delta_c)} \left| \frac{d\delta_c}{dt} \right|, \qquad (9)$$

with R and δ_c on the right hand side written in terms of M and t, respectively, through equations (3) and (4). This formation rate is plotted in Figure 3. No similar quantity is provided by the LCa model.

Finally, by multiplying equation (5) by $\Delta M = M' - M$ we obtain the specific rate at which the mass M of objects is increased at the time t owing to (true) mergers and, by integrating this latter function over M', the instantaneous typical mass increase rate owing to mergers for objects of mass M

$$r^d_{mass}(M,t) = \int_M^\infty \Delta M\, r^d(M \to M',t)\, dM'. \quad (10)$$

Likewise, from equation (6) we can obtain the instantaneous typical mass increase rate for objects of final mass M' owing to accretion and merger,

$$r^c_{mass}(M',t) = \int_0^{M'} M\, r^c(M' \leftarrow M,t)\, dM. \quad (11)$$

As shown in Appendix D, the idea of a *transition* rate, by accretion, similar to those given by equations

(5) or (6) is meaningless because accretion is a continuous instead of a discrete process of mass increase. In contrast, the *mass increase* rate by accretion does make sense. The instantaneous mass accretion rate for objects of mass M follows from the instantaneous scale increase rate of the corresponding peaks as they evolve along continuous and derivable trajectories in the δ vs. R diagram. This latter rate depends on the particular value of the scaled Laplacian x of the peak which is being followed. However, since we are interested in the *typical* mass accretion rate for objects of mass M disregarding any other property we must average over x. This is done in Appendix D. The result is

$$r^a_{mass}(M,t) \approx \frac{1}{\langle x \rangle\, \sigma_2\, R} \frac{dM}{dR} \left| \frac{d\delta_c}{dt} \right|, \qquad (12)$$

with R and δ_c on the right hand side written in terms of M and t, respectively, through equations (3) and (4) and $\langle x \rangle$ the average mentioned after equation (1).

In Figure 4, we plot the total mass increase rate of objects of mass M at t, $r_{mass}(M,t) \equiv r^d_{mass}(M,t) + r^a_{mass}(M,t)$, and compare it with that resulting from the LCa model, i.e., the integral over ΔM of their specific merger rate times the corresponding increase in mass. (Note that this latter integral does not diverge contrary to those involved in the calculation of the total merger and total capture rates.) As expected,
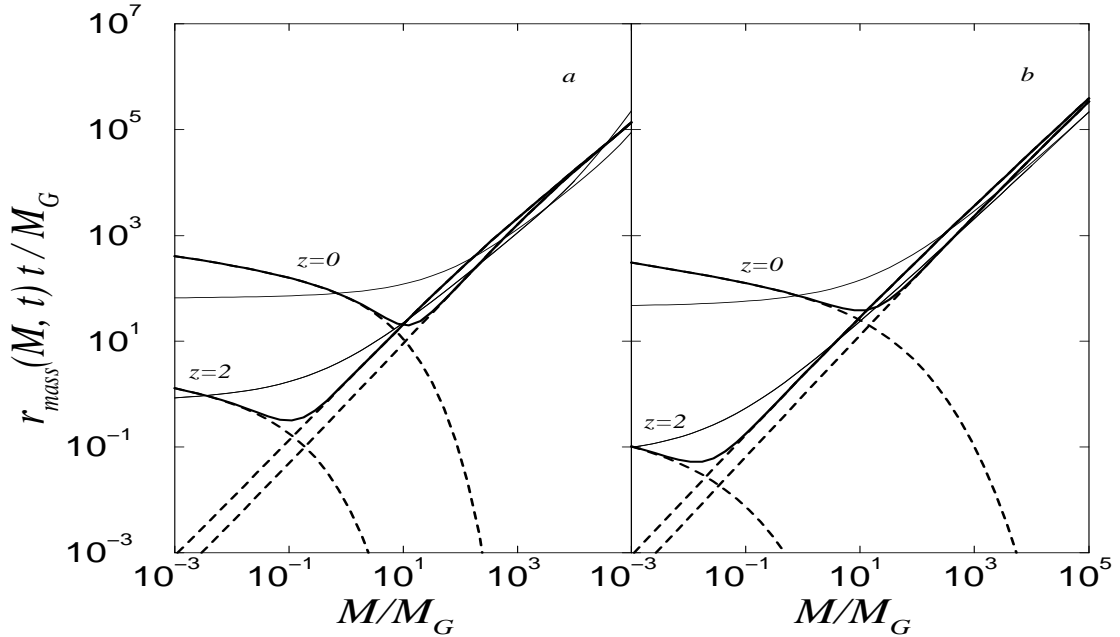
Fig. 4.— Instantaneous total mass increase rate for the same epochs and masses as in Fig. 3. The contributions from mergers and accretion (dominating at the lower and higher mass ends, respectively) are represented in dashed lines. In thin lines the total mass increase rates inferred from LCa merger rates.

the two models show, in this case, similar qualitative behaviors. They only differ quantitatively in the regime dominated by mergers. This departure is just due to the differences in the two merger rates at large $\Delta M/M$ (see Fig.1).

4. GROWTH CHARACTERISTIC TIMES

From the meaning of the global merger rate, equation (7), we have that the density $N_{sur}(t)\, dM$ of objects surviving (i.e., having not merged but just accreted) until the time t from a typical population with masses in the range between M_0 and $M_0 + dM$ at $t_0 < t$ is given by the solution of the differential equation

$$\frac{dN_{sur}}{dt} = -r^d[M(t),t]\, N_{sur}(t), \qquad (13)$$

with initial condition $N_{sur}(t_0) = N(M_0, t_0)$. In equation (13) and hereafter, the function $M(t)$ is the typical mass at t of such accreting objects, approximately given by the solution of the differential equation

$$\frac{dM}{dt} = r^a_{mass}[M(t),t], \qquad (14)$$

with initial condition $M(t_0) = M_0$. Indeed, from equation (12) we have that the typical mass of objects evolving by continuous accretion increases with time according to equation (14). This expression is

only approximated because the average over x leading to equation (12) presumes all objects with the same mass, while the mass $M(t)$ is just the so estimated *typical* mass of objects at t. Hence, the larger the interval of time spent since t_0 when objects have really identical mass the poorer will be the approximation.

The solution of equation (13) is

$$N_{sur}(t) = N(M_0, t_0)\exp\left\{ -\int_{t_0}^{t} r^d[M(t'),t']\, dt' \right\}.$$
(15)

Hence, by defining the typical survival time t_{sur} of objects with masses between M_0 and $M_0 + dM$ at t_0 as the interval of time since t_0 after which their initial density is reduced (owing to mergers) by a factor e, we are led to the equality $t_{sur} = t_d - t_0$, with the destruction time $t_d(M_0, t_0)$ given by the solution of the implicit equation

$$1 = \int_{t_0}^{t_d(M_0,t_0)} r^d[M(t'),t']\, dt'. \qquad (16)$$

In addition, the typical mass accreted by those objects until they merge and disappear is $M[t_d(M_0,t_0)] - M_0$. (Caution: what LCa called survival time rather corresponds to what here is called destruction time; hereafter we assume these authors using the same notation

as ours.)

In a fully similar manner we can infer the typical age of objects with masses in the range between M_0 and M_0+dM at t_0, that is, the typical interval of time since the last merger giving them rise. The density $N_{pre}(t)\,dM$ of these objects pre-existing (i.e., having just accreted matter since then) at a time $t < t_0$ is given by the solution of the differential equation

$$\frac{dN_{pre}}{dt} = r^f[M(t),t]\,N[M(t),t] - r^d[M(t),t]\,N_{pre}(t) \tag{17}$$

with initial condition $N_{pre}(t_0) = N(M_0,t_0)$. The solution of equation (17) is

$$N_{pre}(t) = N(M_0,t_0)\,K^{-1}(t,t_0)$$

$$\times\left\{1 - \int_t^{t_0} dt'\,\frac{N[M(t'),t']}{N(M_0,t_0)}\,r^f[M(t'),t']\,K(t',t_0)\right\} \tag{18a}$$

$$K(t,t_0) = \exp\left\{-\int_t^{t_0} r^d[M(\xi),\xi]\,d\xi\right\} \tag{18b}$$

Thus, by defining the typical age $t_{age}(M_0,t_0)$ of objects with masses between M_0 and M_0+dM at t_0 as the interval of time until t_0 before which their density (owing to their progressive formation) was a factor e smaller, we are led to the equality $t_{age} = t_0 - t_f$, with the formation time $t_f(M_0,t_0)$ given by the solution of the implicit equation

$$1 = \frac{N_{pre}[t_f(M_0,t_0)]}{N(M_0,t_0)}\,\text{e}. \tag{19}$$

And the typical mass accreted by these objects since they formed is $M_0 - M[t_f(M_0,t_0)]$.

Finally, from the typical age and surviving time of objects with masses between M_0 and $M_0 + dM$ at t_0 calculated above, we can readily calculate their typical lifetime or intermerger period. This is simply

$$t_{life}(M_0,t_0) = t_{age}(M_0,t_0) + t_{sur}(M_0,t_0)$$

$$= t_m(M_0,t_0) - t_f(M_0,t_0). \tag{20}$$

And connected with this latter quantity, there is the total mass typically accreted by those objects during their whole life, given by $M[t_m(M_0,t_0)] - M[t_f(M_0,t_0)]$.

In Figures 5 and 6 we plot the typical age and survival time, respectively, of objects at two different epochs as a function of their mass M. For comparison we also plot the corresponding time estimates drawn from LCa model. As mentioned, our natural definitions of the typical age and survival time (i.e., the time spent since the previous merger when the object was formed and the time spent until the next merger when the object will be destroyed, respectively) are not possible in the PS approach not making the distinction between merger and accretion. What LCa call the age and survival time of an object of mass M are the time spent since the mass of some parent object reached half that mass and the time required by the object to be found inside an object of double mass, respectively. (LCa give the time distributions; what are plotted in Figs. 5 and 6 are the corresponding median values.)

Our age shows the same slightly decreasing trend with increasing M and the same strongly increasing trend with increasing time as LCa's. The only difference is that our age is slightly smaller than LCa's. (This trend is reversed at very large masses, for a given redshift, but this is likely due to the poor fit to the PS mass function yielded there by our mass function. Note that a mass of 10^{12} M$_\odot$ at $z = 2$ is comparable to a present mass of two or three orders of magnitude higher for the $n = -2$ power-law and the CDM spectra, respectively.) In contrast, our survival time show a very different, in fact opposite, behavior from LCa's. Our result that the larger the mass of the object, the larger its survival time is just what one would expect in hierarchical clustering distinguishing between accretion and merger. Indeed, at a fixed time, massive objects are rare and they hardly merge, while light objects are more frequent and can more easily merge (see Fig. 3). Moreover, as the time increases objects of a given fixed mass become more and more frequent, hence, their survival time diminishes as also found.

To better understand these comparative behaviors we also plot, in Figures 5 and 6, two new times, hereafter called half-mass-accretion time and double-mass-accretion time, defined à la LCa but in the context of our model. These are the interval of time spent since the mass of an object was typically half its current value and the interval of time required by an object to typically double its mass, respectively. As these new times refer to the typical mass evolution of *given objects* the evolutionary process involved can only be accretion. So they are given by the function $M(t)$ solution of equation (14). It is worthwhile noting that these new times are, in some sense, orthogonal to our age and survival times because the former
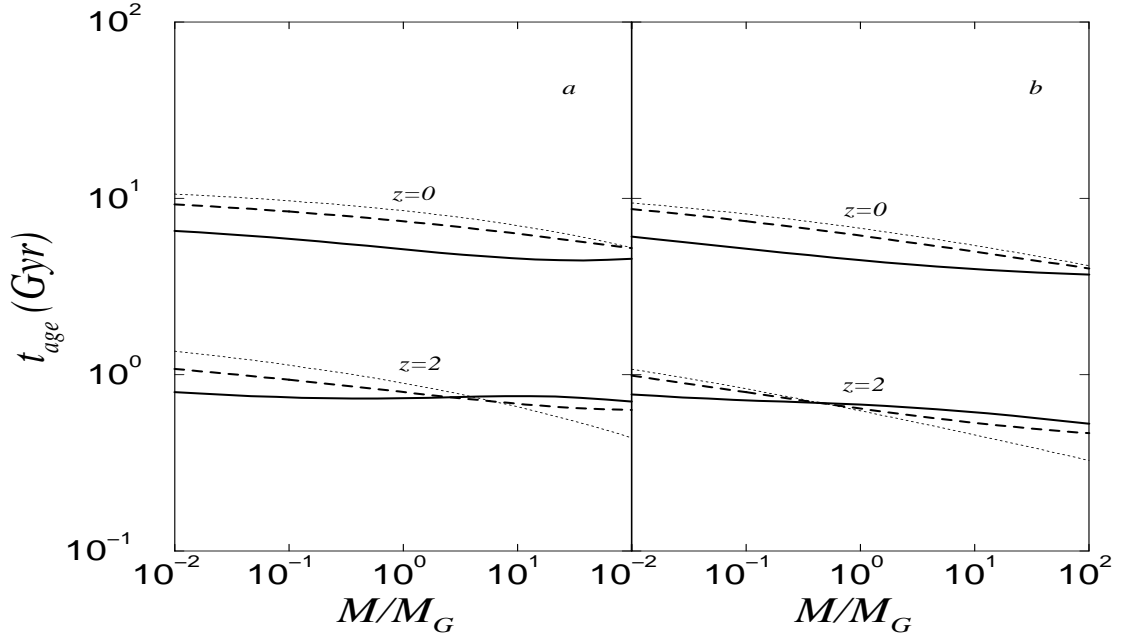
Fig. 5.— Typical age (solid lines) and half-mass-accretion time (dashed lines) of objects of mass M in units of $M_G = 10^{12} \, M_\odot$ for the quoted values of the redshift corresponding to the same cosmogonies (a) and (b) as in the preceding figures. LCa's median age is plotted for comparison (thin dotted lines).
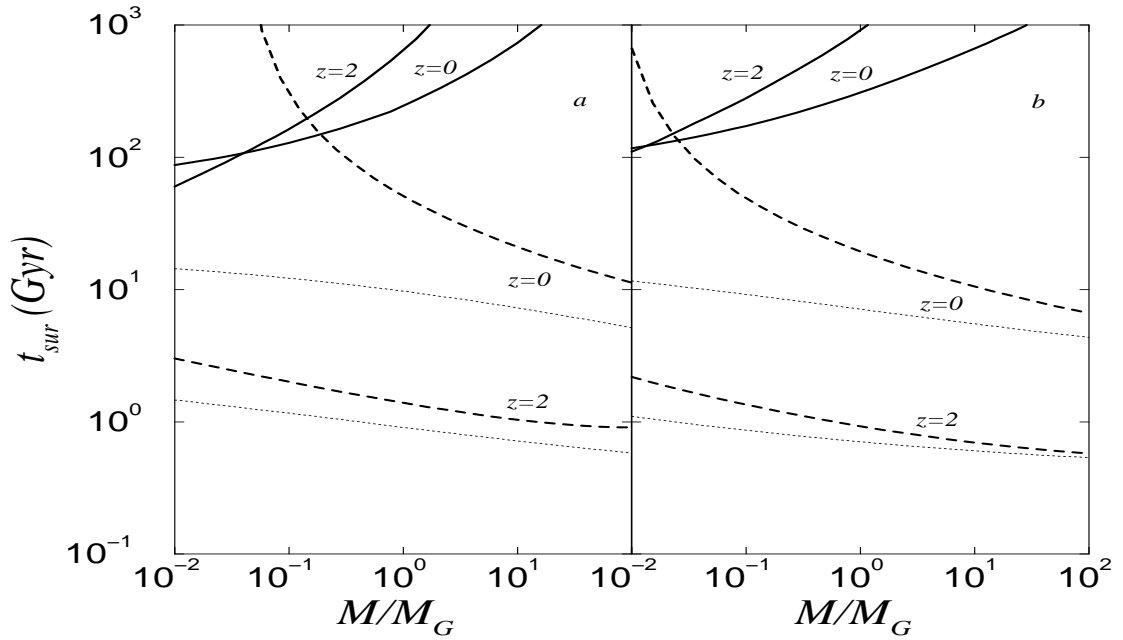

Fig. 6.— Typical survival time (solid lines) and double-mass-accretion time (dashed lines) of objects of mass M in units of $M_G = 10^{12} \, M_\odot$ for the quoted values of the redshift correponding to the same cosmogonies (a) and (b) as in Fig. 4. LCa's median survival time is plotted for comparison (thin dotted lines).

only depend on accretion while the latter mostly do on mergers.

From Figure 5 we see that objects found at a given redshift typically form at a lookback time of the order of (just slightly smaller than) that required for them to accrete half their mass. (Note that our age takes into account that many similar objects that formed earlier or later than this epoch have not survived until the redshift considered.) So mergers play an important role in the past evolution of these objects, but the mass increase experienced by them from the time they typically formed is close to a factor two. It is therefore not surprising that the age estimate à la LCa (accounting for both accretion and mergers) is also close to that time. Notice, however, that the similarity between the half-mass-accretion time and the age as defined in the present work is fortuitous since both time estimates are rather orthogonal. So is the similarity between our age and LCa's.

From Figure 6 we see that the time required by very massive objects at a given epoch to be destroyed is much greater than that required for them to double their mass by accretion, while the opposite is true for very light objects. In other words, there is a gradation from frequent light objects where the mass increase is dominated by mergers to rare massive objects where it is by accretion (see Fig. 3). It is therefore not surprising that the double-mass-accretion time (only accounting for accretion) and the LCa survival time (accounting for accretion and mergers) notably deviate from each other for very light objects while they yield similar results for very massive ones. Note that we should not expect a very good agreement there, either, because the larger the resulting value of the time estimate, the more marked the effects of: 1) the approximation used (eq. [14]) to calculate accretion-times and 2) the difference between the two couples of time estimates; ours reports on "the time required for objects to change their typical mass by a given factor", i.e., they involve averages along R at fixed δ's, while LCa's reports on "the typical time required for objects to change their mass by the same given factor", i.e., they involve averages along δ at fixed R's. In any event, the opposite behavior shown by our survival time as compared to LCa's is also clarified.

5. DISCUSSION AND CONCLUSIONS

The idea that the present clustering model relying on the peak model ansatz can provide a good description of hierachical clustering is at variance with the rather extended opinion that peaks are not good seeds of bound objects. Detailed N-body simulations of the gravitational evolution of individual density maxima seem to show, indeed, that if their initial amplitude is small they can be disrupted before collapsing (van de Weygaert & Babul 1993). While Katz, Quinn, & Gelb (1993) have found that even high amplitude peaks seem to be poor tracers of bound objects, individually as well as statistically. The reason why this would be so is that peaks are not spherical in general, nor are they isolated. So their non-linear evolution will markedly deviate from the spherical approximation at the base of the peak model ansatz. However, any conclusion drawn from the previous N-body simulations dealing with peak statistics is somewhat precipitated because of the unknown effects of the use of unappropriate filtering windows and/or $R(M)$ and $\delta_c(t)$ relations, and the lack of any correction for the cloud-in-cloud effect. In fact, by using the accurate peak-patch formalism, Bond & Myers (1996) have found that there is actually a good correspondence between peaks and high mass objects. This is well understood: the higher the peak relative to the typical density fluctuation on that scale, the more spherical, and the less important the shear caused by the surrounding density fluctuations, particularly if the power spectrum is steep enough ($\sigma_0(M)$ flat enough) to guarantee that nearby density fluctuations on similar scales are typically negligible. This has also been recently confirmed by Bernardeau (1994) who has rigurously shown that the evolution of high amplitude peaks (with $\nu \gtrsim 2$) in a Gaussian random density field is correctly described by the spherical model provided only the logarithmic slope of the power spectrum is smaller than $n = -1$.

A fundamental result of paper I was that the self-consistency of the peak model ansatz, in an extended version presuming some effective distinction between accretion and merger, completely fixes the filtering window and the $R(M)$ and $\delta_c(t)$ relations except for a couple of free parameters governing the dynamics of collapse. After calculating the density of peaks at a fixed δ_c and performing the appropriate correction for the cloud-in-cloud effect we obtained a mass function which is in overall agreement with N-body simulations (since in agreement with the PS mass function) for appropriate values of those parameters.

In the present paper we have calculated the growth rates and characteristic times of relaxed objects predicted by the model. As far as we can tell from the comparison with the LCa model not making the

distinction between merger and accretion, our model yields a reasonably good detailed description of hierarchical clustering for massive objects. Its unsatisfactory behavior at small M is likely due to one practical although rather crude approximation used in paper I to correct for the nesting among peaks at a fixed density contrast. We are currently working on an improved version of the model based on a more accurate correction.

Let us finally insist on the great potential of the approach followed here. The violent relaxation that accompanies the sudden large mass increase of a relaxed system is expected to leave perdurable imprints in its morphological properties as compared to the effects of a quasi-continuous one. A detailed clustering model should therefore make that important distinction. This can be readily carried out in the extreme cases of very large or very small mass captures, but in the intermediate regime it is more problematic. Do we have we the right to assume some definite effective division between tiny and notable captures? Is it unique and where is it? The assumption that there is, indeed, a definite division between accretion and merger yields, in the well motivated framework of the peak theory, one unique, self-consistent, reasonably good clustering model. Thus, the possibility of carrying out such a division in a self-consistent manner seems proved. The comparison with N-body simulations should tell us where the effective division between merger and accretion is, opening in this way the possibility of better describing the growth of cosmic objects and of understanding their observed properties.

A.1. DISAPPEARING PEAKS

From the Taylor series expansion of the gradient of the density contrast smoothed on scale R, $\boldsymbol{\eta}$, around the location $\mathbf{r}_p$ of a neighboring peak we have (to first order in $|\mathbf{r} - \mathbf{r}_p|$)

$$\eta_i \approx -\lambda_i \, (r - r_p)_i, \tag{A1}$$

with $\lambda_i > 0$, the eigenvalues of the second order cartesian derivative tensor ζ_{ij} changed of sign evaluated at $\mathbf{r}_p$ or $\mathbf{r}$. Since there is at most one peak in the neighborhood of any point the density of peaks around a point $\mathbf{r}$ is

$$\delta^{(3)}(\mathbf{r} - \mathbf{r}_p) = |\lambda_1 \lambda_2 \lambda_3| \, \delta^{(3)} \boldsymbol{\eta}. \tag{A2}$$

But η_i and λ_i are random Gaussian variables. So the typical density of peaks on scale R around an arbitrary point $\mathbf{r}$ is given by the mean

$$\langle \delta^{(3)}(\mathbf{r} - \mathbf{r}_p) \rangle = \langle |\lambda_1 \lambda_2 \lambda_3| \, \delta^{(3)} \boldsymbol{\eta} \rangle \tag{A3}$$

for the joint probability of the random variables involved, with λ_i strictly positive, evaluated at the arbitrary point. This is the scheme followed by BBKS for obtaining the density $\mathcal{N}_{pk}(\nu, R) \, d\nu$ of peaks with scaled density contrast ν in an infinitesimal range.

According to the identification criterion among peaks on contiguous scales, a peak at $\mathbf{r}_p$ on scale R will disappear in reaching the scale $R + \Delta R$, with ΔR positive and arbitrarily small, provided only this is the first scale larger than R with no peak in the neighborhood $|\mathbf{r} - \mathbf{r}_p| \leq O(\Delta R)$ of the former peak. From the Taylor series expansion of $\boldsymbol{\eta}$ at the nearest point $\mathbf{r}_{p'}$ with $\boldsymbol{\eta} = 0$ on scale $R + \Delta R$ around the peak $\mathbf{r}_p$ on scale R we have

$$(r_{p'} - r_p)_i \approx \frac{\partial_R \eta_i}{\lambda_i} \, \Delta R. \tag{A4}$$

Thus, on the new scale $R + \Delta R$ there is some peak in the neighborhood $|\mathbf{r} - \mathbf{r}_p| \leq O(\Delta R)$ of the old peak on scale R provided only that all λ_i are of order unity (this also guarantees that all λ_i are positive at $\mathbf{r}_{p'}$ on scale $R + \Delta R$). Strictly, some λ_i could be of order ΔR or smaller, if the corresponding $\partial_R \eta_i$ were too. But the probability for this to happen is negligible as compared to the more general preceding case. Therefore, for that condition to be broken for the first time at $R + \Delta R$, some eigenvector λ_i must become of order ΔR or, equivalently, must vanish at $R \pm \Delta R$.

Consequently, the density of peaks on scale R at an arbitrary point $\mathbf{r}$ disappearing at R' in the neighborhood of R, with $R' > R$, is given by the mean $\langle \delta^{(3)}(\mathbf{r} - \mathbf{r}_p) \rangle$, with all eigenvalues λ_i positive, as in BBKS, *and* the smallest one satisfying the relation

$$R' - R \approx \frac{\lambda_i}{|\partial_R \lambda_i|}. \tag{A5}$$

But condition (A5) implies, at the same time, that the smallest eigenvalue λ_i also vanishes in the neighborhood of $\mathbf{r}_p$ on scale R. Thus, in neglecting second order terms in equation (A1) the term in that component λ_i must be neglected, too. Accordingly, were we interested in calculating the density of peaks disappearing at R' in the neighborhood of R, we should take for that component of $\mathbf{r} - \mathbf{r}_p$, instead of (A1), the relation

$$\partial_R \eta_i - \partial_R \eta_i(\mathbf{r}_p) \approx -\partial_R \lambda_i \, (r - r_p)_i, \tag{A6}$$

with $\partial_R \eta_i(\mathbf{r}_p)$ evaluated at the peak on scale R and the remaining variables evaluated at the arbitrary point $\mathbf{r}$.

Following these schemes it can be shown (Manrique 1995) that the density of peaks at R with ν in an infinitesimal range and all eigenvalues λ_i of zeroth order in ΔR is equal to the total density $\mathcal{N}_{pk}(\nu, R) \, d\nu$ calculated by BBKS to zeroth order in ΔR, while the density of peaks with some λ_i of first order in ΔR contribute to that total density with first order terms in ΔR. Therefore, the density of peaks at δ with R in an infinitesimal range, $N_{pk}(R, \delta) \, dR$, calculated in paper I from the density $\mathcal{N}_{pk}(\nu, R) \, d\nu$ is indistinguishable to leading order in ΔR (or in $\Delta \delta$ for each

given trajectory $\delta(R)$) from the density of peaks at δ robust enough for their continuity to be guaranteed in some neighborhood ΔR (or $\Delta\delta$).

A.2. DENSITIES OF PEAKS WITH SPECIFIC VALUES OF RANDOM VARIABLES

In next Appendixes we will be concerned with densities of peaks with values of the random Gaussian variables with null mean $v_0 \equiv \nu$, $v_1 \equiv x = -(R\sigma_2)^{-1}\partial_R(\sigma_0\nu)$ (this is the same variable x defined in BBKS; see paper I), and

$$v_i \equiv \frac{1}{\sigma_{2i}\,R^{i-1}}\,\partial_R[\sigma_{2(i-1)}\,R^{i-2}\,v_{i-1}] \qquad (i \geq 2) \tag{A7}$$

on scale R in infinitesimal ranges. All these densities can be inferred from the density of peaks with ν and x on scale R in infinitesimal ranges (BBKS)

$$\mathcal{N}_{pk}(\nu, x, R)\,d\nu\,dx = \frac{\exp(-\nu^2/2)}{(2\pi)^2 R_*^3\,[2\pi\,(1-\gamma^2)]^{1/2}}\,f(x)\,\exp\left[-\frac{(x-\gamma\nu)^2}{2(1-\gamma^2)}\right]\,d\nu\,dx \tag{A8}$$

with $f(x)$ a function given by BBKS (eq. [A15]), $\gamma \equiv \sigma_1^2/(\sigma_0\sigma_2)$, and $R_* \equiv \sqrt{3}\sigma_1/\sigma_2$. One must simply apply the recursive relation

$$\mathcal{N}_{pk}(v_0, v_1, ..., v_n, R)\,dv_0\,dv_1\,...dv_n = \mathcal{N}_{pk}(v_0, v_1, ..., v_{n-1}, R)\,dv_0\,dv_1,...dv_{n-1}\mathcal{P}(v_n, R|v_0, v_1, ..., v_{n-1}, R)\,dv_n, \tag{A9}$$

with $\mathcal{P}(w, R|v_0, v_1, ..., v_{n-1}, R)\,dw$ the conditional probability of finding the value of w in an infinitesimal range given that $v_0, v_1, ..., v_{n-1}$ take some given values,

$$\mathcal{P}(w, R|v_0, v_1, ..., v_{n-1}, R)\,dw = \frac{1}{\sqrt{2\pi}\,\sigma_w}\,\exp\left[-\frac{(w-\bar{w})^2}{2\,\sigma_w^2}\right]\,dw \tag{A10a}$$

$$\bar{w} = \mathbf{M}_{w\,v}\,\mathbf{M}_{v\,v}^{-1}\,\mathbf{V}^{\mathrm{T}} \qquad \sigma_w^2 = <w^2> - \mathbf{M}_{w\,v}\,\mathbf{M}_{v\,v}^{-1}\,\mathbf{M}_{w\,v}^{\mathrm{T}}, \tag{A10b}$$

where $\mathbf{M}_{w\,v}$ and $\mathbf{M}_{v\,v}$ are the $1\times n$ and the $n\times n$ correlation matrixes of w with v_i, $\langle w\,v_i\rangle$, and v_i with themselves, $\langle v_i\,v_j\rangle$, respectively, $\mathbf{V}$ stands for the $1\times n$ matrix of components v_i, and T denotes transpose. Note that, although equation (A9) involves, in principle, the conditional probability for peaks, the conditional probability (A10) is for simple points. The reason for this is that the condition for a point to be a peak refers to the values of variables η_i, x, y, and z (see BBKS for the definition of y and z). While none of the variables v_i defined above correlates with any of these variables except for x. Consequently, the conditional probability of finding any specific value of v_i does not depend on whether we are dealing with a peak or simply a point with the same given (positive) value of x.

The correlations among variables v_i take, for the Gaussian window, the general form

$$\langle v_i\,v_j\rangle = (-1)^{i+j+\delta_{0i}+\delta_{0j}}\,\frac{\sigma_{i+j}^2}{\sigma_{2i}\,\sigma_{2j}} \tag{A11}$$

$(i, j \geq 0)$, with δ_{ij} the Krönecker delta. We must remark that, although the notation used in the preceding equations presumes all variables defined on scale R, they are also valid for variables defined on different scales by just appropriately changing the values of the spectral moments involved. In particular, were any variable, say v_i, defined on another scale R', one would be led to just the same expression for the correlations as in equation (A11) but with σ_i and σ_{i+j} replaced by $\sigma_i' \equiv \sigma_i(R')$ and $\sigma_{i+j\ h} \equiv \sigma_{i+j}\{[0.5\,(R^2 + R'^2)]^{1/2}\}$, respectively.

Thus, from equations (A8) to (A11) one can readily calculate the density function $\mathcal{N}_{pk}$ of peaks at a fixed scale R per infinitesimal ranges of ν, x, and any other set of the previous variables v_i $(i \geq 2)$. And by integrating it over variable ν with the constraint (see paper I)

$$\frac{\delta}{\sigma_0} \leq \nu \leq \frac{\delta}{\sigma_0} + x\,\frac{\sigma_2}{\sigma_0}\,R\,\Delta R, \tag{A12}$$

we can infer the density function N_{pk} of peaks at a fixed δ per infinitesimal ranges of R, x, and the same set of variables v_i $(i \geq 2)$. Consequently, such density functions N_{pk} are just equal to the corresponding ones $\mathcal{N}_{pk}$ times $x\,\sigma_2\,R/\sigma_0$.

Moreover, following the same scheme starting from the conditional density analogous to the normal density in equation (A8) but for peaks subject to any given constraint one is led to the same relation as above between couples of conditional density functions $\mathcal{N}_{pk}$ and N_{pk} per infinitesimal ranges of any set of variables v_i $(i \geq 2)$ subject to that constraint.

A.3. NESTING PROBABILITIES

Collapsing clouds associated with non-nested peaks with fixed δ yield a partition of space which makes the mass function of objects at t be correctly normalized (see paper I). This implies that the volume fraction occupied by disjoint backgrounds with δ on filtering scales between R' and $R' + dR'$ or, equivalently, the probability to find a point in any such backgrounds is $M(R')\,\rho^{-1}\,N(R',\delta)\,dR'$. Therefore,

$$P(R',\delta|R,\delta)\,dR' = \frac{M(R')}{\rho}\,N(R',\delta)\,dR'\,\frac{N_{pk}(R,\delta|R',\delta)}{N_{pk}(R,\delta)}. \tag{A13}$$

gives the approximate probability that a typical (non-fissionable) peak with δ on scale R is nested within some non-nested peak with identical density contrast but on a scale between R' and $R' + dR'$ $(R < R')$, hereafter simply called the (differential) nesting probability of a peak. (Remember that the density of peaks that fission in the next dR or $-d\delta$ is a higher order differential.) This was used, in paper I, to derive equation (1). Likewise, the nesting probability of peaks with given values of variables δ, x, and any set of variables v_i $(i = 2, ..., n$ with arbitrary $n)$ is

$$P(R',\delta|R,\delta,x,v_2,...v_n)\,dR' = \frac{M(R')}{\rho}\,N(R',\delta)\,dR'\,\frac{N_{pk}(R,x,v_2,...,v_n,\delta|R',\delta)}{N_{pk}(R,x,v_2,...,v_n,\delta)}, \tag{A14}$$

with $N_{pk}(R,x,v_2,...,v_n,\delta|R',\delta)$ the conditional density function analogous to $N_{pk}(R,\delta|R',\delta)$ in equation (A13) but per infinitesimal ranges of the extra variables $x, v_2, v_3, ..., v_n$. The last factor on the right-hand member of equation (A14) satisfies the relation (see Appendix A.2)

$$\frac{N_{pk}(R,x,v_2,...,v_n,\delta|R',\delta)}{N_{pk}(R,x,v_2,...,v_n,\delta)} = \frac{\mathcal{P}(\nu',R'|\nu,x,v_2,...,v_n,R)\,\mathcal{N}_{pk}(\nu,x,v_2,...,v_n,R)\,x\,\frac{\sigma_2}{\sigma_0}\,R}{\mathcal{P}(\nu',R')\,\mathcal{N}_{pk}(\nu,x,v_2,...,v_n,R)\,x\,\frac{\sigma_2}{\sigma_0}\,R} = \frac{\mathcal{P}(\nu',R'|\nu,x,v_2,...,v_n,R)}{\mathcal{P}(\nu',R')}, \tag{A15}$$

with $\nu' \equiv \delta/\sigma_0'$, $\mathcal{P}(\nu',R'|\nu,x,v_2,...,v_n,R)\,d\nu'$ defined in equation (A10), and $\mathcal{P}(\nu,R)\,d\nu$ the Gaussian probability to find the scaled density contrast on scale R between ν and $\nu + d\nu$. Notice that, for identical reasons as for the conditional probability in equation (A10), the conditional probability given in equation (A14) applies, in fact, to points. Note also that we are using the same convention for the notation of the conditional probabilities as for the density functions: they are denoted by a caligraphic capital p (in contrast with the notation used in BBKS) when they are per infinitesimal range of ν at a fixed R, and by a roman capital p when they are per infinitesimal range of R at a fixed δ. Finally, it is worthwhile mentioning that, contrary to what is suggested by the present notation, we can write ν or δ, indistinctly, when specifying the condition.

From equation (A10), it is clear that the nesting probability given by equations (A14) and (A15) will depend on variables v_i provided only that these variables have non-null correlations with ν (or δ) and x on scale R or any explicit variable correlating with them. This is what happens with the variables v_i defined in equation (A7) involving the different order scale derivatives of the density contrast. (This is readily understood from the Taylor series expansion of ν on scale R' around R: the probability that a point has ν on scale R' depends on the value of all scale derivatives of the density field on scale R.) Thus, to accurately infer the density of nested peaks in any given peak population one must calculate the distribution of these *infinite* variables in that population. The only noticeable exception concerns the *typical population* of peaks. The density of typical peaks at δ with values of v_i $(i = 1, ..., n)$ in infinitesimal ranges, $N_{pk}(R,x,v_2,...v_n,\delta)\,dR\,dx\,dv_2\,...dv_n$, times the nesting probability $P(R',\delta|R,\delta,x,v_2,...v_n)\,dR'$ integrated over any subset of variables v_i coincides with the product of these two

functions without the explicit dependence on the integrated variables, as readily seen from equation (A14) valid for any arbitrary values of the subindexes. In particular, the integral over all v_i for $i \geq 1$ is equal to the product of $N_{pk}(R, \delta) \, dR$ times the reduced nesting probability $P(R', \delta | R, \delta) \, dR'$ given in equation (A13), which justifies this latter expression and equation (1).

The conditional probability $\mathcal{P}(\nu', R' | \nu, x, v_2, v_3, ..., R) \, d\nu'$ extended to the infinite set of variables v_i defined in equation (A7) can be obtained according to equation (A10). After some lenghty algebra using intermediate variables which only correlate with themselves (found by means of the Gramm-Schmidt method) we arrive to the expression

$$\mathcal{P}(\nu', R' | \nu, x, v_2, v_3, ..., R) \, d\nu' \;=\; \frac{1}{\sqrt{2\pi}\,\sigma_{\nu'}} \, \exp\!\left[-\frac{(\nu' - \bar{\nu})^2}{2\,\sigma_{\nu'}^2}\right] d\nu' \tag{A16}$$

with

$$\bar{\nu} \;=\; \alpha_0 \nu + \alpha_1 x + \alpha_2 v_2 + ...,$$

$$\alpha_i = \sum_{j=0}^{\infty} B_j \, \beta_{i\ j-i} \sum_{k=0}^{j} \beta_{k\ j-k} <v_0'\, v_k> \qquad\qquad \sigma_{\nu'}^2 = 1 - \sum_{j=0}^{\infty} B_j \left[\sum_{k=0}^{j} \beta_{k\ j-k} <v_0'\, v_k>\right]^2 \tag{A17}$$

($i \geq 0$), where a prime denotes the scale R', the correlations in angular brackets are given by equation (A11), and coeffecients B_j and $\beta_{k\ j-k}$ are defined as

$$(B_j)^{-1} = 1 + \sum_{k=0}^{j-1} \beta_{k\ j-k}\left(\beta_{k\ j-k} + 2\sum_{l=0}^{j-1-k} \beta_{j-l\ l} <v_k\, v_{j-l}>\right) \qquad\qquad \beta_{i\ j-i} = -\sum_{k=0}^{j-1} C_{i\ j-i}^{k} <v_j\, v_k>, \tag{A18}$$

($j \geq 1$ and $i \neq j$) in addition to $B_0 = 1$ and $\beta_{i\,0} = 1$. In equations (A18) we have used the notation

$$C_{i\,1}^{k} \equiv B_i\, \beta_{k\ i-k} \qquad\qquad C_{i\ j-i}^{k} \equiv \sum_{l=k}^{j-1} B_l\, \beta_{k\ l-k}\, \beta_{i\ l-i} \tag{A19}$$

for $i \leq k$ and $C_{i\,j-i}^{k} \equiv C_{k\,j-k}^{i}$ for $i > k$.

From the general expressions of $\bar{w}$ and σ_w^2, equation (A10), it can be shown, through the intermediate use of variables $\bar{\nu}^{(n)}$ defined as $\bar{\nu}$ but for just the first arbitrary $n+1$ variables v_i, that

$$\langle \bar{\nu}\, v_i \rangle \;=\; \langle \nu'\, v_i \rangle \;\equiv\; (-1)^{i+1+\delta_{0i}}\, \frac{\sigma_{i\,h}^2}{\sigma_0'\,\sigma_{2i}}, \tag{A20}$$

($i \geq 0$) which implies, on its turn, a similar relation for any (finite or infinite) linear combination of v_i, in particular,

$$\langle \bar{\nu}^2 \rangle \;=\; \langle \nu'\, \bar{\nu} \rangle \;=\; 1 - \sigma_{\nu'}^2. \tag{A21}$$

On the other hand, taking into account that the correlation between two variables is equal to the integral of the product of their Fourier transforms taken at $\mathbf{r} = 0$ and defining the new variable $\bar{x}$ as $-(\sigma_2 R)^{-1}\, \partial_R(\sigma_0'\, \bar{\nu})$ we have

$$\langle \bar{x}\, v_i \rangle \;=\; \frac{\sigma_0'}{\sigma_2}\left(\frac{\sigma_{2i+1}^2}{\sigma_{2i}^2} - \frac{1}{R}\, \partial_R\right)\langle \bar{\nu}\, v_i \rangle + (-1)^{\delta_{i\,0}}\, \frac{\sigma_0'\,\sigma_{2(i+1)}}{\sigma_2\,\sigma_{2i}}\, \langle \bar{\nu}\, v_{i+1} \rangle \tag{A22}$$

($i \geq 0$) and

$$\langle \bar{x}\, \bar{\nu} \rangle \;=\; -\frac{\sigma_0'}{\sigma_2}\, \frac{1}{2\,R}\, \partial_R \langle \bar{\nu}^2 \rangle. \tag{A23}$$

By substituting the correlations in equation (A20) into equation (A22) we obtain

$$\langle \bar{x}\, v_i \rangle \;=\; 0 \tag{A24}$$

$(i \geq 0)$ which also implies

$$\langle \bar{x}\, \bar{\nu} \rangle = 0 \qquad\qquad \langle \bar{x}^2 \rangle = 0. \tag{A25}$$

Thus, $\langle \bar{\nu}^2 \rangle$ and $\sigma_{\nu'}$ do not depend on R (see eqs. [A21] and [A23]) which ultimately implies that $\sigma_{\nu'}$ is null and $\langle \bar{\nu}^2 \rangle$ equal to unity. Indeed, according to its definition, equation (A17), $\sigma_{\nu'}$ is null for $R' = R$ and since it does not depend on R it is necessarily null for any value of R'. Then, equation (A16) leads to

$$\mathcal{P}(\nu', R'|\nu, x, v_2, v_3, ..., R)\, d\nu' = \delta(\nu' - \bar{\nu})\, d\nu'. \tag{A26}$$

This result is not surprising since fixing the values of the density contrast and every order scale derivative of it on a given scale R automatically fixes, through the Taylor series expansion of δ as a function of the filtering scale, the value of the density contrast in any other scale R'. Finally, by substituting $\mathcal{P}$ given by equation (A26) into equation (A15) and the latter into (A14) we arrive to the following expression for the nesting probability

$$P(R', \delta|R, \delta, x, v_2, v_3, ...)\, dR' \equiv P(R', \delta|\bar{\nu})\, dR' = \frac{M(R')}{\rho}\, N(R', \delta)\, dR'\, \frac{\delta(\nu' - \bar{\nu})}{\mathcal{P}(\nu')}, \tag{A27}$$

with $\nu' = \delta/\sigma_0'$.

<div align="center">APPENDIX B: NET DENSITY OF PEAKS BECOMING NESTED</div>

The density of peaks at $\delta_f \equiv \delta - \Delta\delta$, with $\Delta\delta$ positive and arbitrarily small, on scales between R_f and $R_f + dR_f$ and variables $x_f, v_{2f}, v_{3f}, ...$ in infinitesimal ranges, which result by continuous evolution (hence, without fissioning) from peaks at δ on scales between R and $R + dR$ and $x, v_2, v_3, ...$ in infinitesimal ranges is

$$N_{pk}^{ev}(R_f, x_f, v_{2f}, v_{3f}, ..., \delta_f)\, dR_f\, dx_f\, dv_{2f} ... = N_{pk}(R, x, v_2, v_3, ..., \delta)\, dR\, dx\, dv_2\, dv_3 ..., \tag{B1}$$

with

$$R_f \approx R + \frac{\Delta\delta}{x\, \sigma_2\, R} \tag{B2}$$

$$x_f \approx x + \left(v_2 \frac{\sigma_4}{\sigma_2} + x \frac{\sigma_3^2}{\sigma_2^2} \right) \frac{\Delta\delta}{x\, \sigma_2} + \left(\sum_i \frac{\partial_R \eta_i}{\lambda_i}\, \partial_i x \right) \frac{\Delta\delta}{x\, \sigma_2\, R}$$

$$v_{2f} \approx v_2 + \left[v_3 \frac{\sigma_6}{\sigma_4} + v_2 \left(\frac{\sigma_5^2}{\sigma_4^2} - \frac{1}{R^2} \right) \right] \frac{\Delta\delta}{x\, \sigma_2} + \left(\sum_i \frac{\partial_R \eta_i}{\lambda_i}\, \partial_i v_2 \right) \frac{\Delta\delta}{x\, \sigma_2\, R} \tag{B3}$$

$$...$$

to first order in $\Delta\delta$. Equation (B1) states that the density $N_{pk}(R, x, v_2, v_3, ..., \delta)\, dR\, dx\, dv_2\, dv_3 ...$ of peaks is conserved through continuous evolution from δ to δ_f. Equations (B2) and (B3) give the shift from δ to δ_f in the values of all the relevant variables. It is important to outline that we need to know, indeed, the values of all variables R and v_i $(i \geq 1)$ at δ_f in order to calculate the density of evolved peaks which are nested since, as explained in Appendix A.3, the nesting probability depends explicitly on all these variables differently distributed in N_{pk}^{ev} than in N_{pk} at δ_f. Equation (B2) arises from the derivative $dR/d\delta$ along continuous peak trajectories, equal to $-(x\, \sigma_2\, R)^{-1}$ (see paper I). While the shift in the variables v_i, equations (B3), is equal to the sum of two terms: one coming from the scale derivative of each particular variable, and a second one coming from the scalar product of its spatial gradient times the shift in position of the new peak relative to the old one (eq. [A4]). However, the nesting probability does not depend on the variables $\partial_R \eta_i$ since these variables do not correlate with $\nu, x, v_2, v_3, ...$ (see Appendix A.3). Thus, in averaging below over all variables, these second terms will contribute with a null mean (the distribution of $\partial_R \eta_i$ for peaks is the same as for arbitrary points). Consequently, we can drop these second terms which is equivalent to taking an effective location for each evolved peak equal to the mean expected value, that is, the same location as the original peak.

From equation (B1) we have that the density of peaks at δ per infinitesimal ranges of R and $x, v_2, v_3,...$ which, after evolving to δ_f, are found to be nested (although not necessarily become nested) into non-nested peaks with scales between R' and $R' + dR'$ ($R \leq R_f < R'$) is

$$N_{pk}^{nest}(R \to R', x, v_2, v_3, ..., \delta \to \delta_f)\, dR\, dR'\, dx\, dv_2\, dv_3\, ...$$

$$= N_{pk}(R, x, v_2, v_3, ..., \delta)\, dR\, dx\, dv_2\, dv_3\, ...\, P(R', \delta_f | R_f, \delta_f, x_f, v_{2f}, v_{3f}, ...)\, dR', \tag{B4}$$

with R_f, x_f, v_{2f}, $v_{3f},...$ on the right hand side in terms of R, x, v_2, $v_3,...$ and $\Delta\delta$ through equations (B2) and (B3), and the specific nesting probability P given in Appendix A.3. To obtain $N_{pk}^{nest}(R \to R', \delta \to \delta_f)\, dR\, dR'$ giving the density of peaks at δ per infinitesimal range of R which after evolving into δ_f are found to be nested into non-nested peaks on scales between R' and $R' + dR'$, we must integrate equation (B4) over variables $x, v_2, v_3, ...$ Given the simple expression of the nesting probability appearing in equation (B4) in terms of the variable $\bar{\nu}$ (eq. [A27]) it is convenient to first transform v_2 and v_3 to $\bar{\nu}$ and $\bar{x}$. This can be done by repeated application of the scheme given in Appendix A.2. Taking into account the correlations (A20) and (A21) and the fact that $\sigma_{\nu'}^2 = 0$ (see the discussion after eq. [A23]) we first obtain

$$N_{pk}(R, x, \bar{\nu}, R', \delta)\, dR\, dx\, d\bar{\nu} = N_{pk}(R, x, \delta)\, dR\, dx\, \frac{e^{-\dfrac{(\bar{\nu} - \bar{\nu}_*)^2}{2\sigma_{\bar{\nu}}^2}}}{\sqrt{2\pi}\, \sigma_{\bar{\nu}}}\, d\bar{\nu}, \tag{B5}$$

with $N_{pk}(R, x, \delta) = \mathcal{N}_{pk}(\nu, x, R)\, x\, \sigma_2\, R/\sigma_0$ in terms of the density function given in equation (A8) and

$$\bar{\nu}_* = \alpha_{\bar{\nu}0}\, \frac{\delta}{\sigma_0} + \alpha_{\bar{\nu}1}\, x \qquad \sigma_{\bar{\nu}}^2 = 1 - \epsilon^2 \left[\frac{(1 - \gamma^2\, r_1)^2}{1 - \gamma^2} + r_1^2 \right] \tag{B6}$$

$$\alpha_{\bar{\nu}0} = \frac{\epsilon\,(1 - \gamma^2\, r_1)}{1 - \gamma^2} \qquad \alpha_{\bar{\nu}1} = -\frac{\epsilon\,\gamma\,(1 - r_1)}{1 - \gamma^2}$$

with $\epsilon \equiv \sigma_{0h}^2/(\sigma_0\, \sigma_0')$ and $r_1 \equiv \sigma_{1h}^2\, \sigma_0^2/(\sigma_{0h}^2\, \sigma_1^2)$ already used in BBKS. Then, from correlations (A24) and (A25) we are led to

$$N_{pk}(R, x, \bar{\nu}, \bar{x}, R', \delta)\, dR\, dx\, d\bar{\nu}\, d\bar{x} = N_{pk}(R, x, \bar{\nu}, R', \delta)\, dR\, dx\, d\bar{\nu}\, \delta(\bar{x})\, d\bar{x}. \tag{B7}$$

This result is well understood. As shown in Appendix A.3, variable $\bar{\nu}$ can only take the same value as ν' and, hence, $\sigma_0'\, \bar{\nu}$ can only depend on R' so that $\bar{x} \equiv -(\sigma_2\, R)^{-1}\, \partial_R(\sigma_0'\, \bar{\nu})$ must be null. Finally, following the same procedure we can infer the density of peaks for the remaining infinite series of variables $v_4, v_5, ...$ in infinitesimal ranges. But this is actually not necessary since the integration of equation (B4) over these latter variables not entering in P is trivial, arriving to

$$N_{pk}^{nest}(R \to R', x, \bar{\nu}, \bar{x}, \delta \to \delta_f)\, dR\, dR'\, dx\, d\bar{\nu}\, d\bar{x} = N_{pk}(R, x, \bar{\nu}, \bar{x}, R', \delta)\, P(R', \delta_f | \bar{\nu}_f)\, dR\, dR'\, dx\, d\bar{\nu}\, d\bar{x}, \tag{B8}$$

with $\bar{\nu}_f = \bar{\nu} - \bar{x}\, \Delta\delta/(x\, \sigma_0')$ and $P(R', \delta_f | \bar{\nu}_f)$ given, for variables with subindex f, by equation (A27). Then, by integrating equation (B8) over $\bar{x}$ and $\bar{\nu}$, which leads to

$$N_{pk}^{nest}(R \to R', x, \delta \to \delta_f)\, dR\, dR'\, dx = \frac{M(R')}{\rho}\, N(R', \delta_f)\, N_{pk}(R, x, \delta | R', \delta_f)\, dR', \tag{B9}$$

taking the Taylor series expansions around δ, keeping first order terms in $\Delta\delta$, and integrating over x (in the positive range) we obtain (see eq. [A13])

$$N_{pk}^{nest}(R \to R', \delta \to \delta_f)\, dR\, dR' = N_{pk}(R, \delta)\, P(R', \delta | R, \delta) \left\{ 1 - \partial_{\delta_f} \ln[N(R', \delta_f)\, N_{pk}(R, \delta | R', \delta_f)] \Big|_{\delta_f = \delta} \Delta\delta \right\} dR\, dR'. \tag{B10}$$

Equation (B10) can also be written, to first order in $\Delta\delta$, as

$$N_{pk}^{nest}(R \to R', \delta \to \delta_f)\, dR\, dR' = N_{pk}(R, \delta)\, dR\, P(R', \delta_f | R, \delta)\, dR', \tag{B11}$$

with

$$P(R', \delta_f | R, \delta)\, dR' \;=\; \frac{M(R')}{\rho}\, N(R', \delta_f)\, dR'\, \frac{N_{pk}(R, \delta | R', \delta_f)}{N_{pk}(R, \delta)} \tag{B12}$$

giving the probability (only for $R' > R$; the case $R' = R$ being excluded) that a (non-fissionable) peak with δ on scale R is located on a disjoint background with δ_f on scales between R' and $R' + dR'$. This is just what one would expect from the same arguments leading to the nesting probability (A13). Notice however that, in contrast to that case, in which $\delta_f = \delta$ so that the corresponding probability could only include the nesting effect, the probability on the left hand side of equation (B12) corresponding to $\delta_f \neq \delta$ will now include not only the nesting of peaks having evolved from R to $R_f < R'$ but also direct evolution of peaks from R to R'. While, by construction, the right-hand member of equation (B10) only includes the former kind of effect. However, as readily seen from a similar development as that leading to N_{pk}^{nest}, the density of peaks with R at δ which directly evolve into the scale R' at δ_f is of high order in $\Delta\delta/(R' - R)$. Therefore, these two expressions coincide, for a given fixed difference $R' - R$, to first order in $\Delta\delta$ as used in equation (B12). (This reasoning shows, in particular, that eq. [B11] is not true in general; it is only approximately satisfied for very small values of $\Delta\delta$. The reason for this is clear. The spatial shift of evolving peaks makes the different order scale derivatives of the density contrast of the evolved peak at δ_f deviate from those of the initial point at the same δ_f for arbitrarily large values of $\Delta\delta$ (see eq. [B3]). The equality only holds to first order in $\Delta\delta$. Therefore, the probability to find a peak with R at δ located in a disjoint background with R' at δ_f is different, in general, from the probability that the corresponding evolved peak at δ_f is located in that background. These two probabilities only approximately coincide for very small $\Delta\delta$.)

We are now ready to calculate the net density of peaks with δ on scales between R and $R + dR$ *becoming nested* into non-nested peaks with $\delta_f \equiv \delta - \Delta\delta$ on scales between R' and $R' + dR'$, $N^d(R \to R', \delta \to \delta_f)\, dR\, dR'$. (Superindex d stands for destruction since these filtering events correspond to true mergers, hence the destruction, of objects.) To do this we must compute the density of peaks which, after evolving (without fissioning) from δ to δ_f, turn out to be nested into non-nested peaks with such larger scales minus the density of (non-fissionable) peaks at δ which are nested in the ancestors of those disjoint backgounds at δ_f. (These ancestors concern, in principle, any kind of evolution, although the contribution of direct evolution can be ignored in practice since being of higher order in $\Delta\delta$). This correction can be readily performed following the same procedure as leading to equation (1). The result is the integral equation

$$N^d(R \to R', \delta \to \delta_f) = N_{pk}^{nest}(R \to R', \delta \to \delta_f) \int_R^{R'} dR''\, \frac{M(R'')}{\rho}\, N^d(R'' \to R', \delta \to \delta_f)\, N_{pk}(R, \delta | R'', \delta) \tag{B13}$$

for the unknown function $N^d(R \to R', \delta \to \delta_f)$ in terms of $N_{pk}^{nest}(R \to R', \delta \to \delta_f)$ (eq. [B10]) and $N_{pk}(R, \delta | R'', \delta)$ (see eq. [1]).

But we do not need to solve equation (B13). Given the meaning of N^d and N, we have $N^d(R'' \to R', \delta \to \delta) \equiv N(R', \delta)\, \delta(R' - R'')$. (Notice that R'' reaches the value R' inside the integral of eq. [B13].) And since $N_{pk}^{nest}(R \to R', \delta \to \delta)$ is equal to $N_{pk}(R, \delta)\, P(R', \delta | R, \delta)$ (see eq. [B10]) we are led to $N^d(R \to R', \delta \to \delta) \equiv 0$ for $R < R'$ as in the present case. Thus, equation (B13) reduces to a simple relation between first order terms in $\Delta\delta$. By dividing this relation by $\Delta\delta$ we are led (see eqs. [B10] and [A13]) to the Volterra type integral equation of the second kind

$$\mathbf{N}^d(R \to R', \delta) = -\frac{M(R')}{\rho}\, \partial_{\delta_f}[N(R', \delta_f)\, N_{pk}(R, \delta | R', \delta_f)]\Big|_{\delta_f = \delta} - \int_R^{R'} dR''\, \frac{M(R'')}{\rho}\, \mathbf{N}^d(R'' \to R', \delta)\, N_{pk}(R, \delta | R'', \delta) \tag{B14}$$

whose solution gives the wanted net density $\mathbf{N}^d(R \to R', \delta)\, dR\, dR'\, d\delta$ of peaks at δ with scales between R and $R + dR$ becoming nested (merging) into non-nested peaks with scales between R' and $R' + dR'$ in the next $-d\delta$. Equation (B14) can be solved numerically by iteration from the initial approximate solution $-M(R')\, \rho^{-1}\, \partial_{\delta_f}[N_{pk}(R', \delta)\, N(R, \delta | R', \delta)]\big|_{\delta_f = \delta}$. Actually, this latter function is a very good approximation to the wanted solution (with an error of less than 10% for the whole range of R' in the least favorable case of R close to

the lower limit of validity) for all power spectra analyzed. Thus, we can simply take

$$\mathbf{N}^d(R \to R', \delta) \approx - \frac{M(R')}{\rho} \, \partial_{\delta_f}[N(R', \delta_f) \, N_{pk}(R, \delta|R', \delta_f)]\Big|_{\delta_f = \delta} \tag{B15}$$

with $\partial_\delta N(R', \delta)$ given by numerical solution of the new Volterra type integral equation

$$
\begin{aligned}
\partial_\delta N(R', \delta) &= \left[\partial_\delta N_{pk}(R', \delta) - \int_{R'}^{\infty} dR'' \, \frac{M(R'')}{\rho} \, N(R'', \delta) \, \partial_\delta N_{pk}(R', \delta|R'', \delta)\right] \\
&\quad - \int_{R'}^{\infty} dR'' \, \frac{M(R'')}{\rho} \, \partial_\delta N(R'', \delta) \, N_{pk}(R', \delta|R'', \delta)
\end{aligned} \tag{B16}
$$

resulting from differentiation of equation (1).

APPENDIX C: NET DENSITY OF NON-NESTED APPEARING PEAKS

The net density of peaks appearing at δ with scales between R and $R+dR$ and variables x, v_2, v_3,... in infinitesimal ranges is equal to the density of (non-fissionable) peaks with these characteristics minus the density of peaks at δ with identical characteristics arising by continuous evolution (hence, without fissioning) from peaks at $\delta_i \equiv \delta + \Delta\delta$, with $\Delta\delta$ positive and arbitrarily small. Therefore, the net density of *non-nested* peaks appearing (forming) at δ with scales between R and $R + dR$ is

$$\mathbf{N}^f(R, \delta, x, v_2, v_3, ...) \, \Delta\delta \, dR \, dx \, dv_2 \, dv_3 .... = \left[1 - P(R, \delta, x, v_2, v_3, ...)\right]$$

$$\times \left[N_{pk}(R, x, v_2, v_3, ..., \delta) - N_{pk}^{ev}(R, x, v_2, v_3, ..., \delta)\right] dR \, dx \, dv_2 \, dv_3 ..., \tag{C1}$$

with $P(R, \delta, x, v_2, v_3, ...)$ the integral (over $R' > R$) of the differential nesting probability given by equation (A14) and $N_{pk}^{ev}(R, x, v_2, v_3, ..., \delta) \, dR \, dx \, dv_2 \, dv_3 ...$ the density of evolved peaks, equal to

$$N_{pk}^{ev}(R, x, v_2, v_3, ..., \delta) \, dR \, dx \, dv_2 \, dv_3 ... = N_{pk}(R_i, x_i, v_{2i}, v_{3i}, ..., \delta_i) \, dR_i \, dx_i \, dv_{2i} \, dv_{3i}... \tag{C2}$$

(Superindex f stands for formation since these filtering events correspond to the formation of new objects.) Equation (C2) is but equation (B1) for the present notation, with the relations between variables with and without subindex i equal to the inverse of those given in equations (B2) and (B3). Note that, for the same reasons as in Appendix B, we are forced to follow the evolution of the whole infinite set of variables $x, v_2, v_3, ...$

From equations (C1) and (C2) we have

$$\mathbf{N}^f(R, \delta, x, v_2, v_3, ...) \, \Delta\delta \, dR \, dx \, dv_2 \, dv_3 ... = \left[1 - P(R, \delta, x, v_2, v_3, ...)\right]$$

$$\times \left[N_{pk}(R, x, v_2, v_3, ..., \delta) - N_{pk}(R_i, x_i, v_{2i}, ..., \delta_i)|J|\right] dR \, dx \, dv_2 \, dv_3 ... \tag{C3}$$

with J the Jacobian of the transformation from variables with subindex i to variables without subindex. Thus, by integrating equation (C3) over all intermediate variables we obtain

$$\mathbf{N}^f(R, \delta) \, \Delta\delta \, dR = \left\{ N(R, \delta) - \int [1 - P(R, \delta, x, v_2, ...)] \, N_{pk}(R_i, x_i, v_{2i}, ..., \delta_i) \right.$$

$$\left. \times \left[1 - \frac{\Delta\delta}{x_i R \sigma_2} \left(R\frac{\sigma_3^2}{\sigma_2^2} - \frac{1}{R}\right)\right] dx_i \, dv_{2i} ... \right\} dR. \tag{C4}$$

In deriving equation (C4) we have used the fact that $N_{pk}(R, \delta)$ times the probability that a peak with δ on scale R be non-nested, $1 - P(R, \delta)$, is just equal to $N(R, \delta)$ (eqs. [A13] and [1]). We have also transformed the variables

at δ in the integral on the right hand side to variables at δ_i, which balances the Jacobian $|J|$, and then changed the differential dR_i by dR according to equation (B2).

For identical reasons as in Appendix B, it is convenient to express N_{pk} and the nesting probability P in equation (C4) in terms of variables $\bar{\nu}$ and $\bar{x}$ instead of v_2 and v_3. After this substitution, the integration over variables $v_4, v_5, \ldots$ becomes trivial, and equation (C4) can be written as

$$\mathbf{N}^f(R, \delta)\,\Delta\delta\,dR = \left\{ N(R, \delta) - \int [1 - P(R, \delta, \bar{\nu})]\, N_{pk}(R_i, x_i, \bar{\nu}_i, \bar{x}_i, \delta_i) \left[1 - \frac{\Delta\delta}{x_i R\sigma_2} \left(R\frac{\sigma_3^2}{\sigma_2^2} - \frac{1}{R} \right) \right]\, dx_i\, d\bar{\nu}_i\, d\bar{x}_i \right\} dR,$$
(C5)

where R_i and δ_i are functions of R and x_i (see eq. [B2]) and δ, respectively. The integral involving only the density N_{pk} is straightforward, whereas that involving the product $P\,N_{pk}$ can be calculated following the same steps as in Appendix B. By integrating over the variables $\bar{x}_i$, $\bar{\nu}_i$ we are led to

$$\mathbf{N}^f(R, \delta)\,\Delta\delta\,dR = \left\{ N(R, \delta) - \int_0^\infty [1 - P(\delta|\delta_i, x_i, R_i)]\, N_{pk}(R_i, x_i, \delta_i) \left[1 - \frac{\Delta\delta}{x_i R\sigma_2} \left(R\frac{\sigma_3^2}{\sigma_2^2} - \frac{1}{R} \right) \right]\, dx_i \right\} dR, \quad \text{(C6)}$$

with $P(\delta_f|\delta, x, R)\,dR$ the integral over R' of the probability $P(R', \delta|\delta_i, x_i, R_i)\,dR$ that a (non-fissionable) peak with R_i and x_i at δ_i is located in a disjoint background with δ on scales between R' and $R' + dR'$ (eq. [B12] but with the extra dependence on x_i through the factor $N_{pk}(R_i, \delta_i, x_i|R, \delta)/N_{pk}(R_i, \delta_i, x_i)$). Then taking the Taylor series expansions around δ, keeping first order terms in $\Delta\delta$, integrating over x_i (in the positive range), and finally dividing by $-\Delta\delta$ and taking the limit $\Delta\delta \to 0$ we arrive to the relation

$$\mathbf{N}^f(R, \delta)\,d\delta\,dR = \left\{ \partial_\delta N_{pk}(R, \delta) - \int_R^\infty dR'\, \frac{M(R')}{\rho}\, N(R', \delta)\, \partial_{\delta_i} N_{pk}(R, \delta_i|R', \delta) \Big|_{\delta_i = \delta} \right.$$

$$\left. - \partial_R \mathcal{N}_{pk}(\delta, R) + \int_R^\infty dR'\, \frac{M(R')}{\rho}\, N(R', \delta)\, \partial_R \mathcal{N}_{pk}(\delta, R|\delta, R') \right\} d\delta\,dR.$$
(C7)

For simplicity, we will take

$$\mathbf{N}^f(R, \delta)\,d\delta\,dR \approx [\partial_\delta N_{pk}(R, \delta) - \partial_R \mathcal{N}_{pk}(\delta, R)]\, d\delta\,dR$$
(C8)

which is a reasonable approximation (with an error of less than 15 % and 20 % for scales within the range of validity of the model in the cases of the CDM and the $n = -2$ power law spectra, respectively) to the exact relation (C7). Note that, given the meaning of the densities $N_{pk}(R, \delta)\,dR$ and $\mathcal{N}_{pk}(\delta, R)\,d\delta$ (see paper I), expression (C8) is just the density of peak trajectories leaving minus entering in the square infinitesimal area of the δ vs. R diagram, equal to the density of peak trajectories appearing minus disappearing (i.e., fusioning minus fissioning) in that infinitesimal area. That is, the approximation leading to (C8) is but the neglect of the correction for nesting.

APPENDIX D: MASS ACCRETION RATE

From equation (B2) we have that the mass accreted from δ to $\delta_f \equiv \delta - \Delta\delta$, with $\Delta\delta$ positive and arbitrarily small, by any non-nested peak with initial scale between R and $R + dR$ and variable x in an infinitesimal range is

$$\Delta M = \frac{dM}{dR}\, \frac{1}{x\,\sigma_2\,R}\, \Delta\delta.$$
(D1)

The density of such accreting non-nested peaks is

$$N^a(R, x, \delta)\,dR\,dx = N(R, x, \delta)\,dR\,dx.$$
(D2)

In writing equation (D2) we have taken into account that the density of non-nested peaks not accreting because merging from δ to $\delta - \Delta\delta$ is a higher order correction (see the discussion leading to eq. [B14]). Therefore, by dividing the density (D2) by $N(R, \delta)\,dR$ we obtain the conditional probability $p(x, R|\delta, R)\,dx$ that an accreting non-nested

peak at δ with scale R has the appropriate value of x in order to increase its mass by ΔM given by equation (D1) in the passage from δ to $\delta - \Delta\delta$. From equation (1) but for peaks with variable x in an infinitesimal range (or, equivalenly, from eq. [A14], after integrating the product $N_{pk} P$ over the remaining variables) this conditional probability takes the form

$$p(x, R|\delta, R) \, dx = \frac{dx}{N(R, \delta)} \left[N_{pk}(R, x, \delta) - \int_R^\infty dR' \, \frac{M(R')}{\rho} \, N(R', \delta) \, N_{pk}(R, x, \delta|R', \delta) \right], \tag{D3}$$

with $N_{pk}(R, x, \delta) = \mathcal{N}_{pk}(\delta, x, R) \, x \, \sigma_2 \, R$ and $N_{pk}(R, x, \delta|R', \delta) = \mathcal{N}_{pk}(\delta, x, R|\delta, R') \, x \, \sigma_2 \, R$ in terms of the analogous density functions calculated by BBKS (see eq. [A8] and paper I).

By changing variable x into $M' = M + \Delta M$ we can compute the instantaneous accretion rate of objects of mass M per specific range of mass M' of the final object similar to the merger rate (6). By doing so we arrive to the fact that this transition rate is identically null. The reason for this is that, as mentioned in Appendix B, the density of peaks with scales between R and $R + dR$ at δ which directly evolve into peaks with scales between R' and $R' + dR'$ at δ_f is of higher order than one in $\Delta\delta = \delta_f - \delta$. This result is well understood. From the viewpoint of the accreting object, the process is not a transition between two different masses but as a continuous mass increase. Consequently, no discrete increment ΔM can be achieved in the limit $\Delta\delta \to 0$. Of course, such a continuous evolution of the accreting object during the small interval Δt necessarily causes (is made at the expense of) the merger (capture) of a number of tiny objects which do make a finite transition in mass. And it is taking the limit for vanishing Δt of the change in the number density of these latter objects that one obtains a non-vanishing accretion(+merger) rate (see § 3).

In any event, there is no problem, even from the viewpoint of accreting objects, in obtaining the instantaneous *mass accretion* rate at t for objects of mass M. Equation (D1) tells us that the instantaneous mass increase rate, by accretion, for objects arising from peaks with δ and the specific value of x is

$$\frac{dM}{dt} = \frac{dM}{dR} \frac{1}{x \, \sigma_2 \, R} \left| \frac{d\delta_c}{dt} \right|. \tag{D4}$$

Therefore, the instantaneous mass accretion rate at t for objects of mass M (disregarding any other particularity) is the average of the specific rate (D4) for the probability function (D3) with R and $\delta = \delta_c$ expressed in terms of M and t through equations (3) and (4). By performing this average we arrive to

$$r_{mass}^a(M, t) = \frac{dM}{dR} \frac{1}{x_{eff} \, \sigma_2 \, R} \left| \frac{d\delta_c}{dt} \right|, \tag{D5}$$

with

$$\frac{1}{x_{eff}} = \frac{1}{\langle x \rangle} \left[1 + \int_R^\infty dR' \, \frac{M(R')}{\rho} \, N(R', \delta) \left(1 - \frac{\langle x \rangle}{\widetilde{\langle x \rangle}} \right) \frac{N_{pk}(R, \delta|R', \delta)}{N(R, \delta)} \right]. \tag{D6}$$

Since $1/\langle x \rangle$ is a very good approximation to $1/x_{eff}$ (with an error of less than 5% for masses within the range of validity of the model) we will simply take

$$r_{mass}^a(M, t) \approx \frac{dM}{dR} \frac{1}{\langle x \rangle \, \sigma_2 \, R} \left| \frac{d\delta_c}{dt} \right|. \tag{D7}$$

REFERENCES

Bahcall, N.A., & Cen, R. 1993, ApJ, 407, L49

Bardeen, J.M., Bond, J.R., Kaiser, N., & Szalay, A.S. 1986, ApJ, 304,15 (BBKS)

Bernardeau, F. 1994, ApJ, 427, 51

Bond, J.R., Cole, S., Efstathiou, G., Kaiser, N. 1991, ApJ, 379, 440

Bond, J.R., & Myers, S.T. 1996, ApJS, 103, 41

Bower, R.J., 1991, MNRAS, 248, 332

Carlberg R.G., & Couchman, H.M.P. 1989, ApJ, 340, 47

Cole, S., & Lacey, C. 1996, MNRAS, in press (astro-ph 9510147)

Efstathiou, G., Frenk, C.S., White, S.D.M., & Davis, M. 1988, MNRAS, 235, 715

Efstathiou, G., & Rees, M.J. 1988, MNRAS, 230, 5p

González-Casado, G., Mamon, G., & Salvador-Solé, E. 1994, ApJ, 433, L61

González-Casado, G., Serna, A., Alimi, J.-M., & Salvador-Solé, E. 1996, ApJ, in preparation

Katz, N., Quinn, T., & Gelb, J.M. 1993, MNRAS, 265, 689

Kauffmann, G. 1994, MNRAS, 274, 153

Kauffmann, G., & White, S.D.M., 1993, MNRAS, 261, 921

Kauffmann, G., & White, S.D.M., & Guiderdoni, B. 1993, MNRAS, 264, 201

Kitayama, T., & Suto, Y. 1996, MNRAS, submitted (astro-ph 9602076)

Lacey, C., & Cole, S. 1993, MNRAS, 262, 627 (LCa)

Lacey, C., & Cole, S. 1994, MNRAS, 271, 676 (LCb)

Manrique, A. 1995, PhD Thesis

Manrique, A., & Salvador-Solé, E. 1995, ApJ, 453, 6 (paper I)

Nolthenius, R., & White, S.D.M. 1987, MNRAS, 225, 505

Press, W.H., & Schechter, P. 1974, ApJ, 187, 425

Richstone, D., Loeb, A., & Turner E. 1992, ApJ, 393, 477

Salvador-Solé, E., & Manrique, A. 1994, in Clusters of Galaxies, eds. F. Durret, A. Mazure & J. Tran Thanh Van (Gif-sur-Yvette: Frontières), p. 297

Salvador-Solé, E., & Manrique, A. 1995, ApLC, in press

Toth, G., Ostriker, J.P. 1992, ApJ, 389, 5

van de Weygaert, R., & Babul, A. 1994, ApJ, 425, L59

White, S.D.M., Efstathiou, G., & Frenk, C.S. 1993, MNRAS, 265, 727